Issues in Zonal Flows and Drift Wave Turbulence

P.H. Diamond

CMTFO and CASS, UCSD; USA

WCI Center for Fusion Theory, NFRI; Korea

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Outline

A) A Look Back and A Look Around: Basic Ideas of the Drift Wave-Zonal Flow System

B) A Look Ahead: Current Applications to Selected Problems of Interest

A) A Look Back and A Look Around

Basic Ideas of the Drift Wave – Zonal Flow System

i) Physics of Zonal Flow Formation

ii) Shearing Effects on Turbulence Transport

iii) Closing the Feedback Loops: Predator(s) Meet Prey

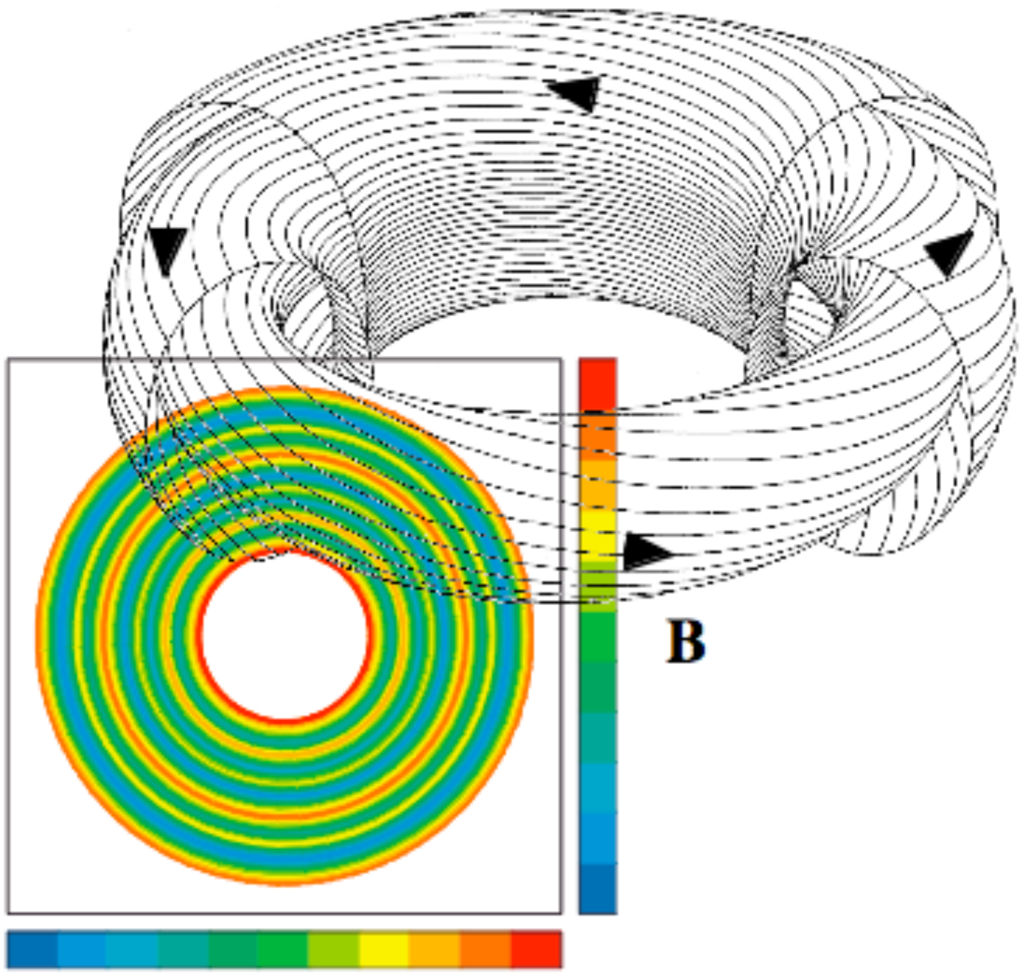
"The difference between an idea and a theory is that the first can generate a call to action and the second cannot."

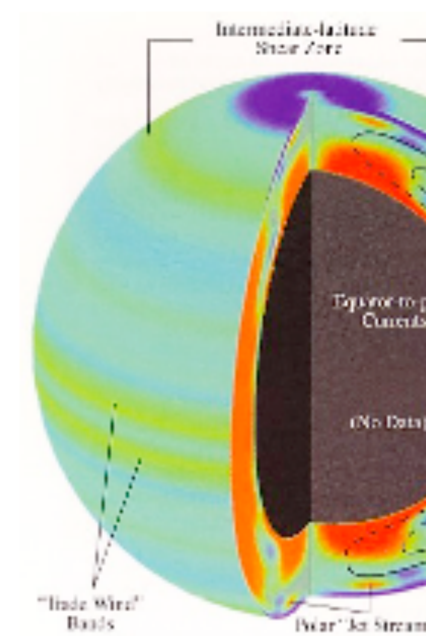
— Stanley Fish

Preamble I

Tokamaks

planets





The sun

Zonal Flows:

$$m = n = 0$$

finite k_r

potential fluctuations

Preamble II

→ Re:Plasma?

→ 2 Simple Models

a.) Hasegawa-Wakatani (collisional drift inst.)

b.) Hasegawa-Mima (DW)

a.) $\mathbf{V} = \frac{c}{B}\hat{z} \times \nabla\phi + \mathbf{V}_{pol}$

$\rightarrow m_s$

$L > \lambda_D$ → $\nabla \cdot \mathbf{J} = 0$ → $\nabla_\perp \cdot \mathbf{J}_\perp = -\nabla_\| J_\|$

$J_\perp = n|e|V_{pol}^{(i)}$

$J_\| : \eta J_\| = -(1/c)\cancel{\partial_t A_\|} - \nabla_\| \phi + \nabla_\| p_e$

e.s.

b.) $dn_e/dt = 0$

→ $$\frac{dn_e}{dt} + \frac{\nabla_\| J_\|}{-n_0|e|} = 0$$

n.b.

MHD: $\partial_t A_\|$ v.s. $\nabla_\| \phi$

DW: $\nabla_\| p_e$ v.s. $\nabla_\| \phi$

So H-W

$$\rho_s^2 \frac{d}{dt} \nabla^2 \hat{\phi} = -D_{\|} \nabla_{\|}^2 (\hat{\phi} - \hat{n}/n_0) + \nu \nabla^2 \nabla^2 \hat{\phi}$$

$$\frac{d}{dt} n - D_0 \nabla^2 \hat{n} = -D_{\|} \nabla_{\|}^2 (\hat{\phi} - \hat{n}/n_0)$$

$D_{\|} k_{\|}^2 / \omega$ is key parameter

n.b. $\quad \mathrm{PV} = n - \rho_s^2 \nabla^2 \phi \qquad \frac{d}{dt}(\mathrm{PV}) = 0$

→ total density

b.) $D_{\|} k_{\|}^2/\omega \gg 1 \rightarrow \hat{n}/n_0 \sim e\hat{\phi}/T_e \qquad (m, n \neq 0)$

$$\frac{d}{dt}(\phi - \rho_s^2 \nabla^2 \phi) + v_* \partial_y \phi = 0$$

→ H-M

n.b. $\quad \mathrm{PV} = \phi - \rho_s^2 \nabla^2 \phi + \ln n_0(x)$

n.b. Zonal Flows: $\rho_s^2 \frac{d}{dt} \nabla^2 \phi = -\mu \nabla^2 \phi + \nu \nabla^2 \nabla^2 \phi$

An infinity of models follow:

- MHD: ideal ballooning
  resistive → RBM

- HW + $A_{||}$: drift - Alfven

- HW + curv. : drift - RBM

- HM + curv. + Ti: Fluid ITG

- gyro-fluids

- GK

N.B.: Most Key advances appeared in consideration of simplest possible models

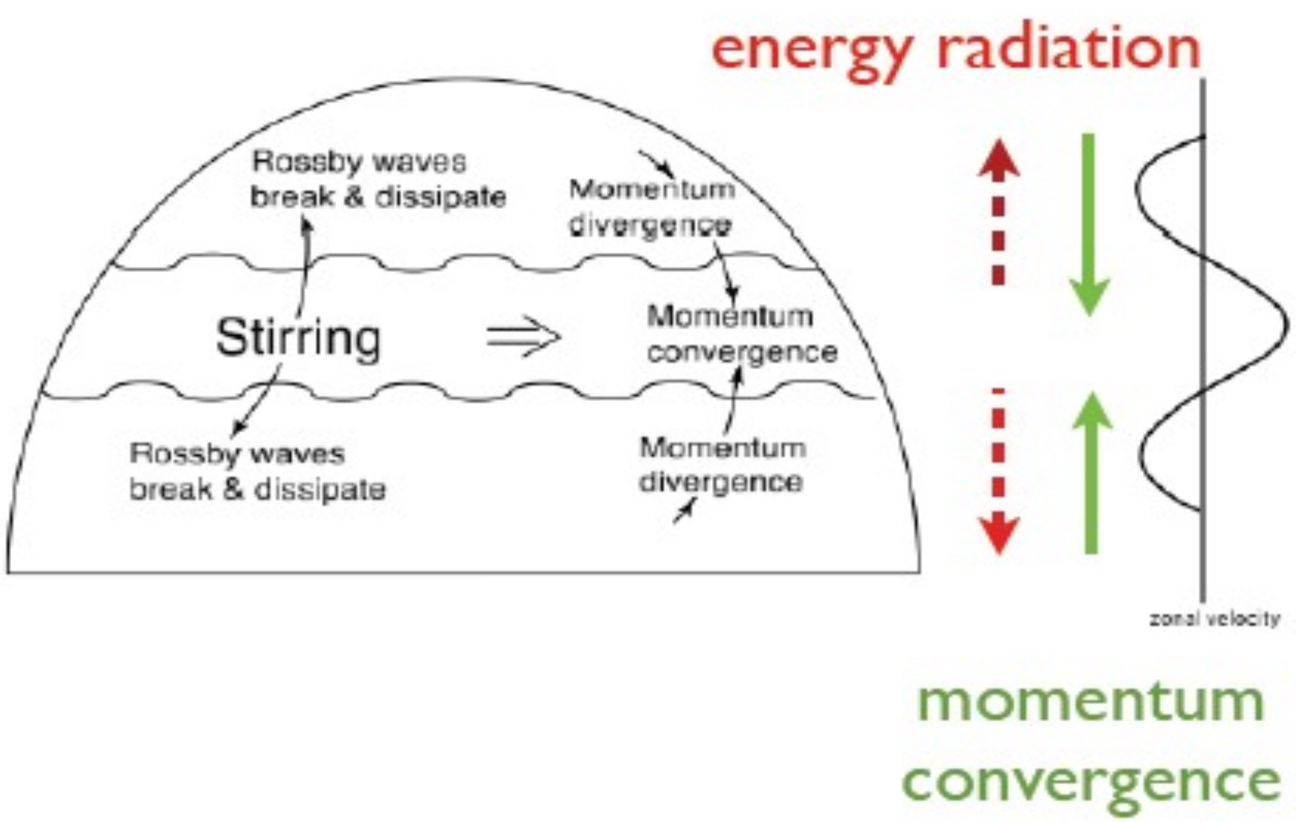
Preamble II

- What is a Zonal Flow?
  - $n = 0$ potential mode; $m = 0$ (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric *ExB* shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport ($n = 0$)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

Heuristics of Zonal Flows a):

Simplest Possible Example: Zonally Averaged Mid-Latitude Circulation

- classic GFD example: Rossby waves + Zonal flow (c.f. Vallis '07, Held '01)
- Key Physics:



Rossby Wave:

$$\omega_k = -\frac{\beta k_x}{k_\perp^2}$$

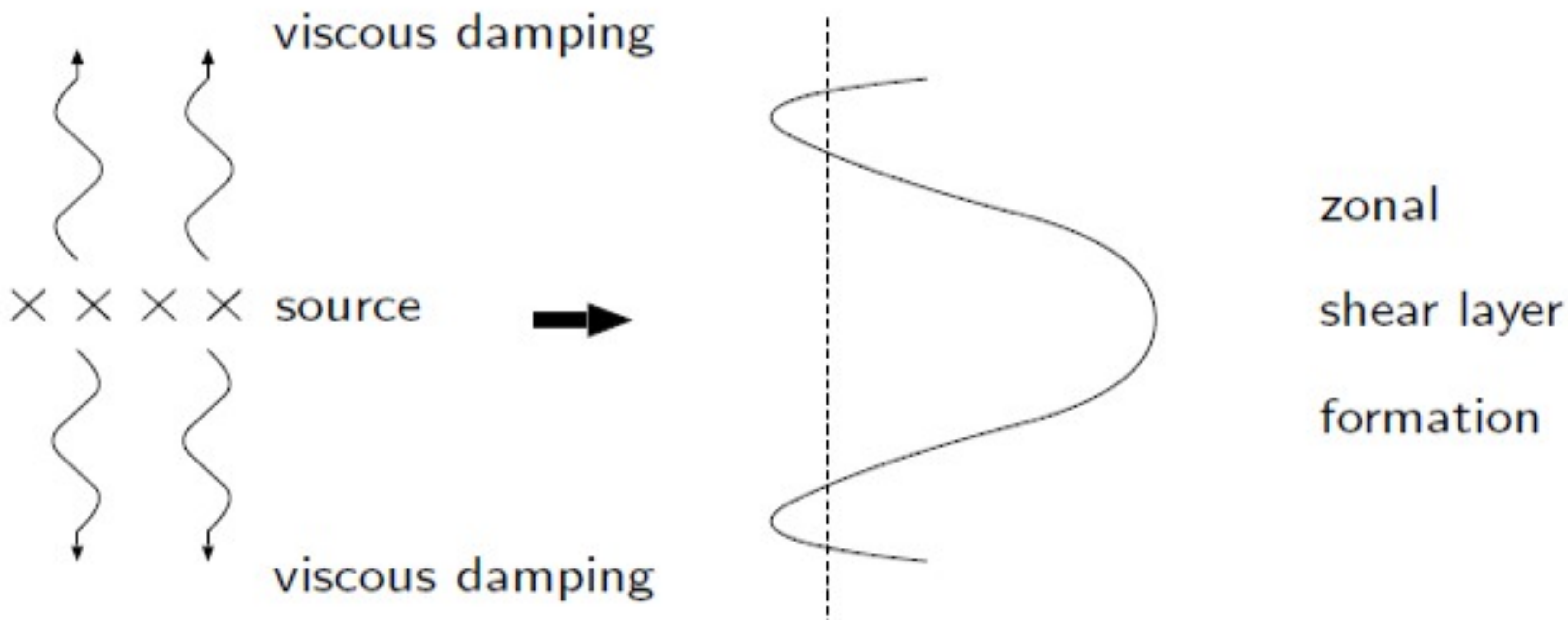
$$v_{gy} = 2\beta \frac{k_x k_y}{k_\perp^2} \qquad \langle \tilde{v}_y \tilde{v}_x \rangle = \sum_k -k_x k_y \left|\hat{\phi}_{\vec{k}}\right|^2$$

$$\therefore \; v_{gy} v_{phy} < 0$$

→ Backward wave!

⇒ Momentum convergence at stirring location

- …"the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into this region." (I. Held, '01)
- Outgoing waves $\Rightarrow$ incoming wave momentum flux

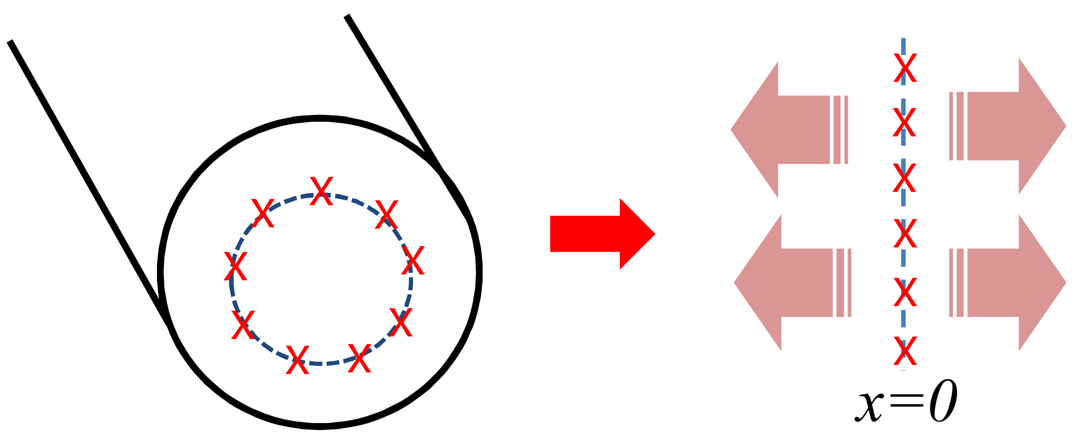


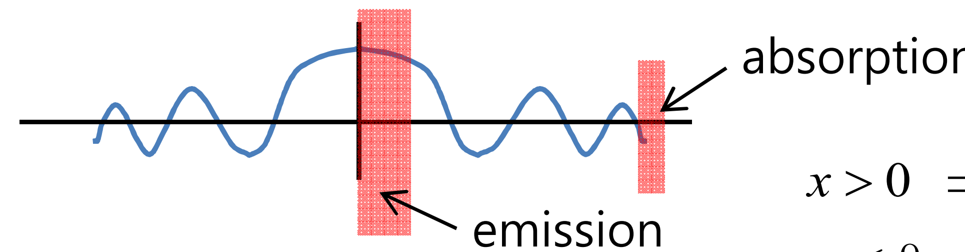
- Local Flow Direction (northern hemisphere):
  - eastward in source region
  - westward in sink region
  - set by $\beta > 0$
  - Some similarity to spinodal decomposition phenomena
    $\rightarrow$ both `negative diffusion' phenomena

Preamble VI

MFE perspective on Wave Transport in DW Turbulence

- localized source/instability drive
  - couple to damping ↔ outgoing wave i.e. Pearlstein-Berk eigenfunction





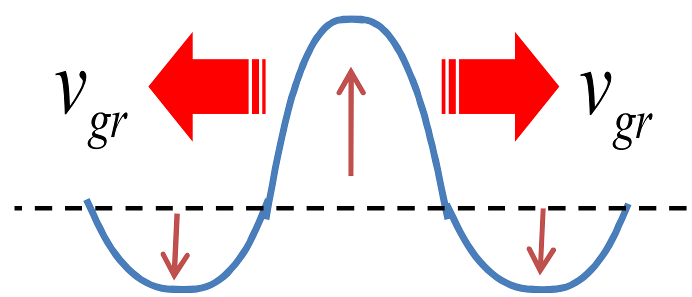
$x > 0 \;\Rightarrow\; v_{gr} > 0$

$v_* < 0 \;\Rightarrow\; v_{gr} > 0$

– $v_{gr} = -2\rho_s^2 \dfrac{k_\theta k_r v_*}{(1+k_\perp^2\rho_s^2)^2}$

– $\langle v_{rE} v_{\theta E}\rangle = -\dfrac{c^2}{B^2} |\phi_{\vec{k}}|^2 \, k_r k_\theta < 0$

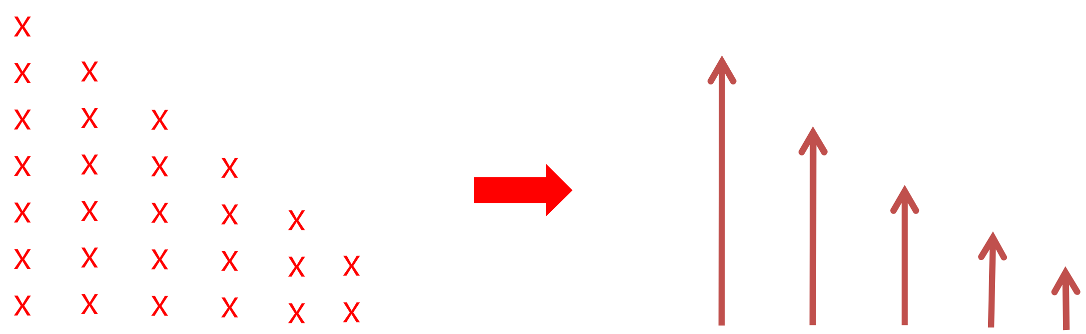
- outgoing wave energy flux → incoming wave momentum flux
  → counter flow spin-up!



- zonal flow layers form at excitation regions

Preamble VII

- So, if spectral intensity → net shear flow → mean shear formation



$$S_r = v_{gr}\varepsilon = -\frac{2k_\theta k_r v_* \rho_s^2}{(1+k_\perp^2\rho_s^2)^2}\varepsilon$$

$$\langle \tilde{v}_r \tilde{v}_\theta \rangle \approx -\sum_k k_r k_\theta |\phi_{\vec{k}}|^2$$

- Reynolds stress proportional to radial wave energy flux S via mode propagation physics (Diamond, Kim '90)
- Equivalently: $\frac{\partial}{\partial t}E + \nabla\cdot\vec{S} + (\omega_k \operatorname{Im}\varepsilon_k)E = 0$ (Wave Energy Theorem) E = wave energy

  ∴ Wave dissipation coupling sets Reynolds force at stationarity
- Interplay of drift wave and ZF drive originates in mode dielectric
- Generic mechanism…

Preamble VIII

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

(M. McIntyre)

- G.C. ambipolarity breaking → polarization charge flux → Reynolds force
  - Polarization charge ➡ $-\boxed{\rho^2}\nabla^2\phi = \boxed{n_{i,GC}(\phi) - n_e(\phi)}$
    polarization length scale *ion GC* *electron density*
  - so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle \neq 0$ ⬌ 'PV transport'
    polarization flux → What sets cross-phase?
  - If 1 direction of symmetry (or near symmetry):

$$-\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$$

(Taylor, 1915)

$-\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$ ➡ Reynolds force ➡ Flow

Additional Comments I

- Heresy: Rigorous "inverse cascade" concept does not seem fundamental?! Well known that Z.F.'s develop on scale of flux, spectral inhomogeneity, without clear \`scale separation'
- Indeed, forward potential enstrophy cascade seems more fundamental to PV mixing and zonal flow formation
- c.f. S. Tobias, et. al. ApJ 2011 → ZF's appear without higher order cumulants

Additional Comments II

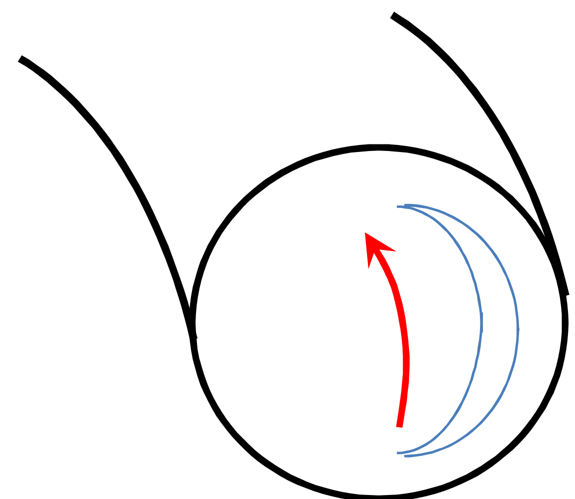
- Mechanisms for PV mixing: A Partial List
  - direct dissipation, as by $\nu\nabla^2$
  - forward potential enstrophy cascade → couple to $\nu\nabla^2$
  - local: wave absorption at critical layers, where $\omega = k_y \langle V_x(y) \rangle$

    global: overlap of neighboring 'cat's eyes' islands

    → streamline stochastization
  - linear and non-linear wave-fluid element interaction (akin NLLD)

Damping

- Yet more: $\frac{\partial}{\partial t}\langle v_\perp \rangle = -\partial_r \langle \tilde{v}_{rE}\tilde{v}_{\perp E} \rangle - \underbrace{\boxed{\gamma_d \langle v_\perp \rangle}}_{\textit{damping}} + \mu \nabla_r^2 \langle v_\perp \rangle$
- Reynolds force opposed by flow damping
- Damping:
  - Tokamak ➡ $\gamma_d \sim \gamma_{ii}$
    - trapped, untrapped friction
    - no Landau damping of (0, 0)
  - Stellerator/3D ➡ $\gamma_d \leftrightarrow NTV$
    - damping tied to non-ambipolarity, also
    - largely unexpored

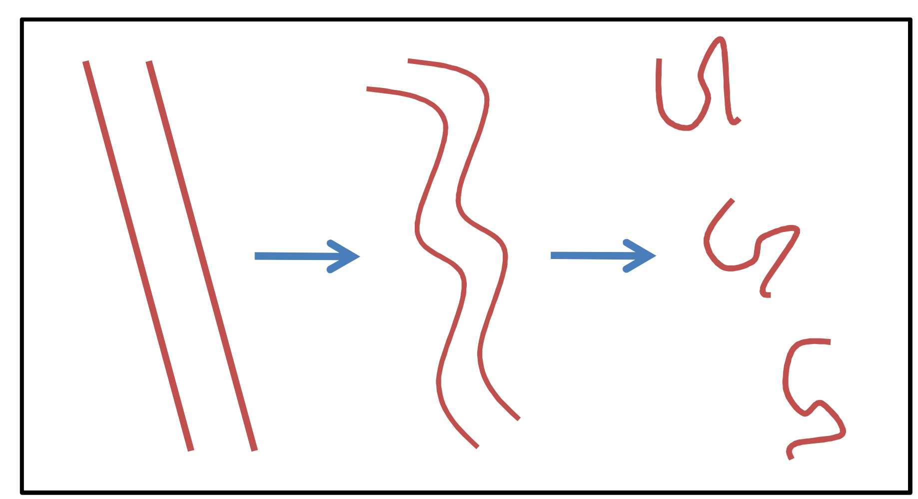


- Weak collisionality → nonlinear damping

  → tertiary → 'KH' of zonal flow →

  but: magnetic shear! (cost/benefit?)

  → other mechanisms?



Zonal Flows

- Potential vorticity transport and momentum balance
  - Example: Simplest interesting system → Hasegawa-Wakatani
    - Vorticity: $\frac{d}{dt}\nabla^2\phi = -D_{\parallel}\nabla_{\parallel}^2(\phi - n) + D_0\nabla^2\nabla^2\phi$
    - Density: $\frac{dn}{dt} = -D_{\parallel}\nabla_{\parallel}^2(\phi - n) + D_0\nabla^2 n$

$$\left\{ \begin{array}{ll} D_0 & \text{classical, feeble} \\ \mathrm{Pr} = 1 & \text{for simplicity} \end{array} \right.$$

  - Locally advected PV: $q = n - \nabla\phi^2$
    - PV: charge density $\left\{ \begin{array}{l} n \rightarrow \text{guiding centers} \\ -\nabla\phi^2 \rightarrow \text{polarization} \end{array} \right.$
    - conserved on trajectories in inviscid theory $\boxed{dq/dt=0}$
    - PV conservation → $\left. \begin{array}{l} \text{Freezing-in law} \\ \text{Kelvin's theorem} \end{array} \right\} \rightarrow$ Dynamical constraint

Zonal Flows, cont'd

- Potential Enstrophy (P.E.) balance

$d\langle q^2\rangle / dt = 0$ P.E. flux small scale dissipation $\langle\ \rangle \to$ coarse graining

$$\text{LHS} \Rightarrow \frac{d}{dt}\langle\tilde{q}^2\rangle \equiv \partial_t\langle\tilde{q}^2\rangle + \partial_r\langle\tilde{V}_r\tilde{q}^2\rangle + D_0\langle(\nabla\tilde{q})^2\rangle$$

$\text{RHS} \Rightarrow \text{P.E. evolution} - \langle\tilde{V}_r\tilde{q}\rangle\langle q\rangle' \Rightarrow$ P.E. Production by PV mixing / flux

- PV flux: $\langle\tilde{V}_r\tilde{q}\rangle = \langle\tilde{V}_r\tilde{n}\rangle - \langle\tilde{V}_r\nabla^2\tilde{\phi}\rangle$; but : $\langle\tilde{V}_r\nabla^2\tilde{\phi}\rangle = \partial_r\langle\tilde{V}_r\tilde{V}_\theta\rangle$

∴ P.E. production directly couples particle transport drive and Reynolds force

- Fundamental Stationarity Relation for Vorticity flux

$$\langle\tilde{V}_r\nabla^2\tilde{\phi}\rangle = \langle\tilde{V}_r\tilde{n}\rangle + (\delta_t\langle\tilde{q}^2\rangle)/\langle q\rangle'$$

Reynolds force Relaxation Local PE decrement

∴ Reynolds force locked to driving flux and P.E. decrement; transcends quasilinear theory

Zonal Flows, cont'd

- Momentum Theorem (Charney, Drazin 1960, et. seq. P.D. et. al. '08)

$$\partial_t\{(GWMD)+\langle V_\theta\rangle\}=-\langle\tilde{V}_r\tilde{n}\rangle-\delta_t\langle\tilde{q}^2\rangle/\langle q\rangle'-\nu\langle V_\theta\rangle$$

drag

driving flux

Local P.E. decrement

GWMD = Generalized Wave Momentum Density; $-\langle\tilde{q}^2\rangle/\langle q\rangle'$ (pseudomomentum)

- "Non-Acceleration Theorem"

$$\partial_t\{(GWMD)+\langle V_\theta\rangle\}=-\langle\tilde{V}_r\tilde{n}\rangle-\delta_t\langle\tilde{q}^2\rangle/\langle q\rangle'-\nu\langle V_\theta\rangle$$

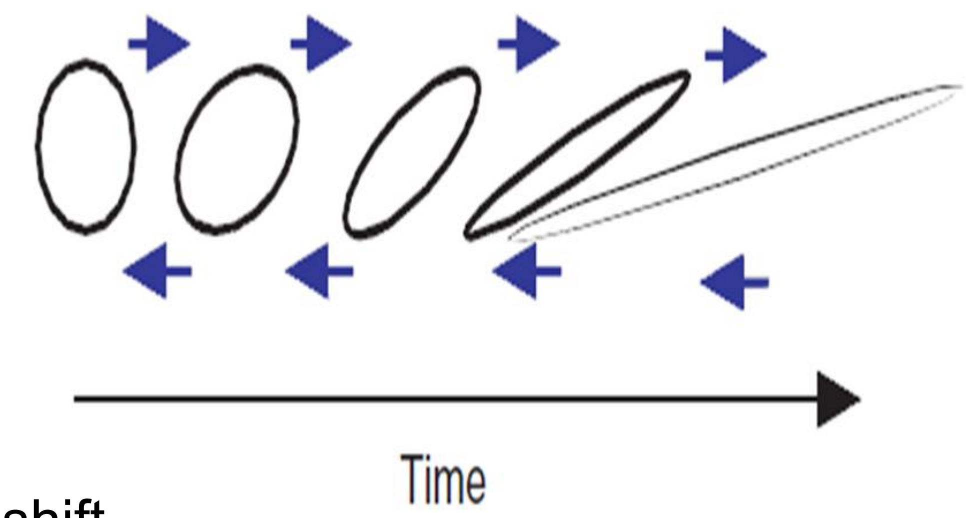
  – Absent $\langle\tilde{V}_r\tilde{n}\rangle$ driving flux; $\delta_t\langle\tilde{q}^2\rangle$ — local potential enstrophy decrement

  → cannot $\left\{\begin{array}{l}\text{accelerate}\\ \text{maintain}\end{array}\right\}$ Z.F. with stationary fluctuations!

- Fundamental constraint on models of stationary zonal flows! ↔ need explicit connection to relaxation, dissipation

Shearing I

- Coherent shearing: (Kelvin, G.I. Taylor, Dupree'66, BDT'90)
  - radial scattering + $\langle V_E \rangle' \rightarrow$ hybrid decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1/\tau_c$
  - shaping, flux compression: Hahm, Burrell '94



- Other shearing effects (linear):

Response shift
and dispersion

  - spatial resonance dispersion: $\omega - k_\| v_\| \Rightarrow \omega - k_\| v_\| - k_\theta \langle V_E \rangle' (r - r_0)$ → cross phases!
  - differential response rotation → especially for kinetic curvature effects

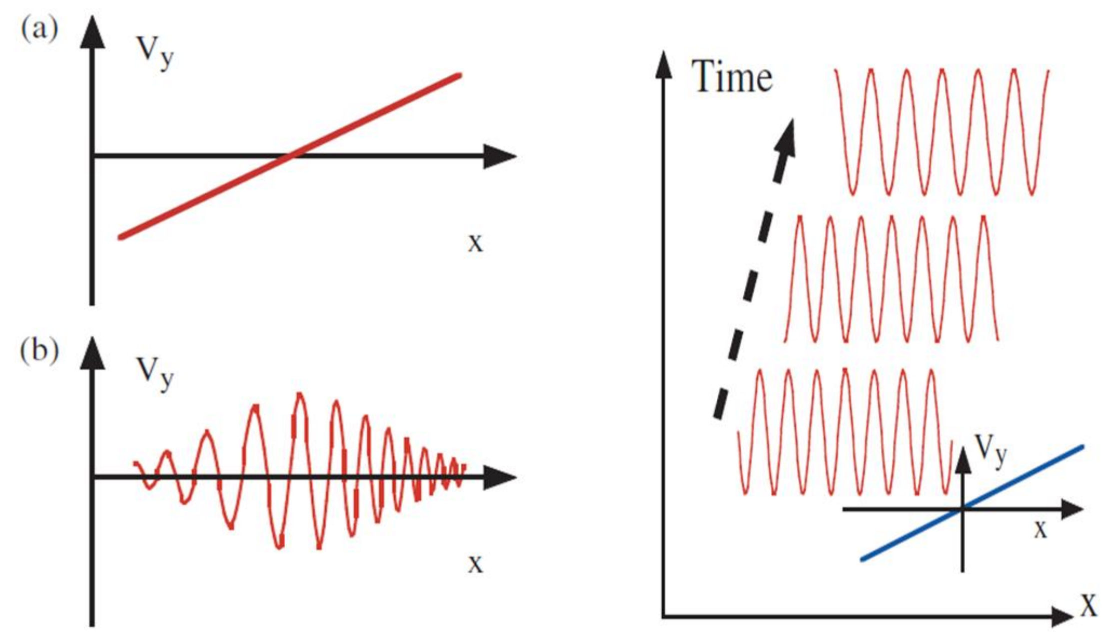
→ N.B. Caveat: Modes can adjust to weaken effect of external shear

(Carreras, et. al. '92; Scott '92)

Shearing II

Mechanisms to Amplify Shears

- Consider:
  - Radially propagating wave packet
  - Adiabatic shearing field



$$\frac{d}{dt}k_r = -\frac{\partial}{\partial r}\left(\omega + k_\theta \left\langle V_{E,ZF} \right\rangle\right) \quad \Rightarrow \quad \left\langle k_r^2 \right\rangle \uparrow$$

$$\omega_{\vec{k}} = \frac{\omega_*}{1 + k_\perp^2 \rho_s^2} \quad \downarrow$$

- Wave action density $N_k = E(k)/\omega_k$ adiabatic invariant

$\therefore$ $E(k)\uparrow$ $\Rightarrow$ flow energy increases, due Reynolds work $\Rightarrow$ flows amplified (cf. energy conservation)

Shearing III

Formally ....

- Zonal Shears: Wave kinetics (Zakharov et. al.; P.D. et. al. '98, et. seq.)

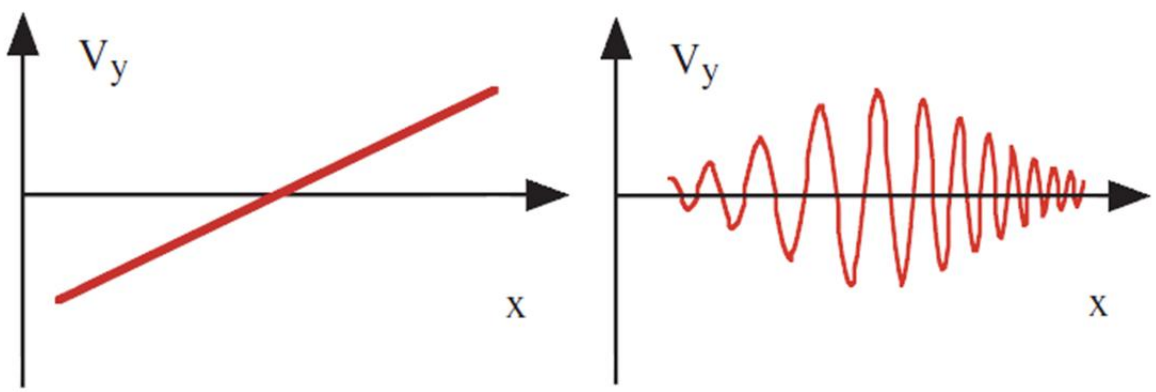
  Coherent interaction approach (L. Chen et. al.)

- $dk_r / dt = -\partial(\omega + k_\theta V_E)/\partial r\,;\ V_E = \langle V_E \rangle + \widetilde{V}_E$

  Mean shearing $: k_r = k_r^{(0)} - k_\theta V_E' \tau$

  Zonal Random shearing $: \langle \delta k_r^2 \rangle = D_k \tau$

  $$D_k = \sum_q k_\theta^2 \left| \widetilde{V}'_{E,q} \right|^2 \tau_{k,q}$$



– Wave ray chaos (not shear RPA) underlies $D_k \rightarrow$ induced diffusion

– Induces wave packet dispersion

– Applicable to ZFs and GAMs

– $\tau_{k,q} \equiv$ coherence time of wave packet ***k*** with shear mode ***q***

- Mean Field Wave Kinetics

$$\frac{\partial N}{\partial t} + (\vec{V}_{gr} + \vec{V}) \cdot \nabla N - \frac{\partial}{\partial r}(\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial \vec{k}} = \gamma_{\vec{k}} N - C\{N\}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N \rangle = \gamma_{\vec{k}} \langle N \rangle - \langle C\{N\} \rangle}$$

Zonal shearing

Shearing IV

- Energetics: Books Balance for Reynolds Stress-Driven Flows!
- Fluctuation Energy Evolution – Z.F. shearing

$$\int d\vec{k}\,\omega\left(\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle\right) \Rightarrow \frac{\partial}{\partial t}\langle \varepsilon\rangle = -\int d\vec{k}\, V_{gr}(\vec{k}) D_{\vec{k}} \frac{\partial}{\partial k_r}\langle N\rangle \qquad V_{gr} = \frac{-2k_r k_\theta V_* \rho_s^2}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

Point: For $d\langle\Omega\rangle / dk_r < 0$, Z.F. shearing damps wave energy

N.B.: For zonal shears, $N \sim \Omega$

- Fate of the Energy: Reynolds work on Zonal Flow

Modulational Instability

$$\partial_t \delta V_\theta + \partial\left(\delta\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle\right) / \partial r = -\gamma \delta V_\theta$$

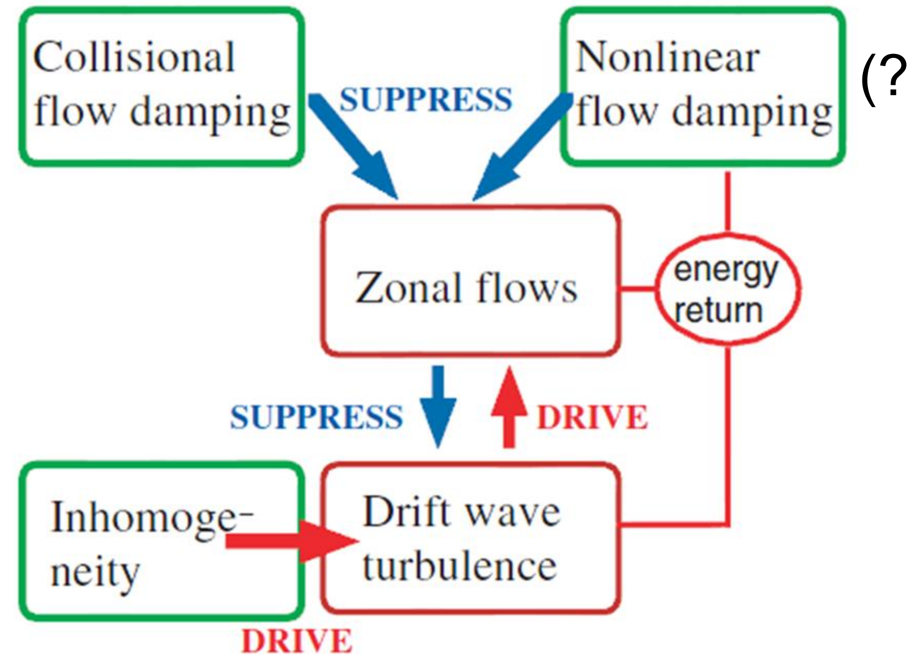
$$\delta\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \sim \frac{k_r k_\theta \delta\Omega}{(1+k_\perp^2\rho_s^2)^2}$$

N.B.: Wave decorrelation essential: Equivalent to PV transport/mixing (c.f. Gurcan et. al. 2010)

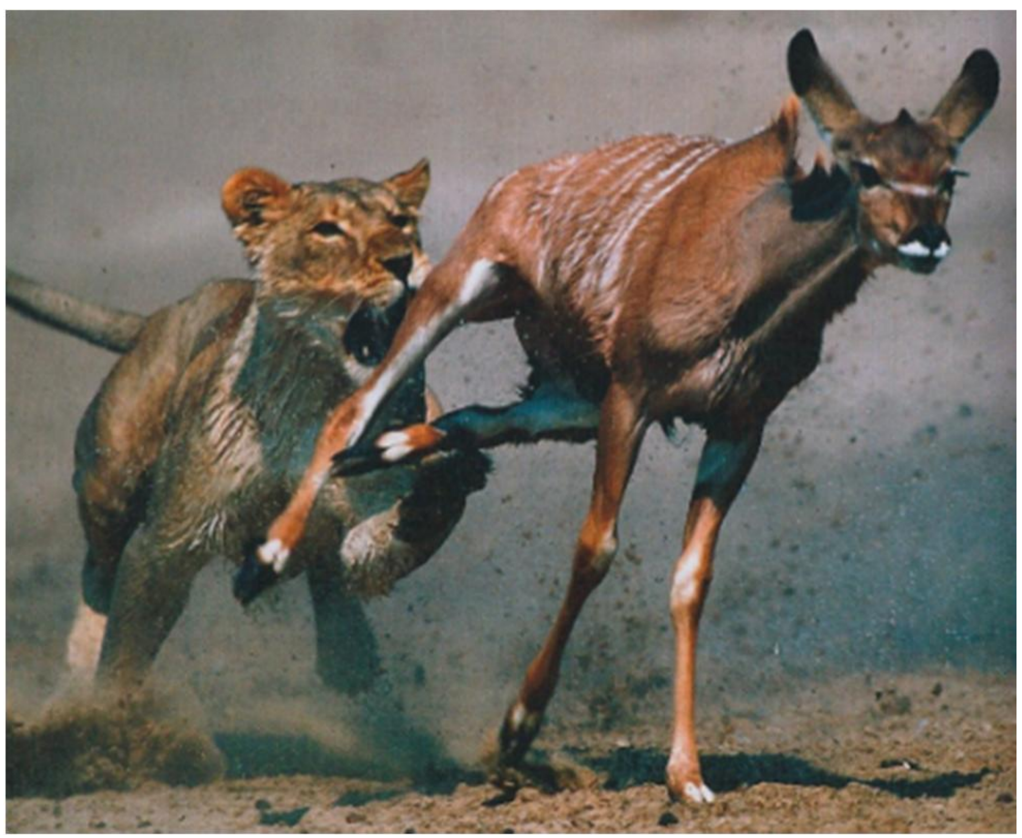
- Bottom Line:
  - Z.F. growth due to shearing of waves
  - “Reynolds work” and “flow shearing” as relabeling → books balance
  - Z.F. damping emerges as critical; MNR ‘97

Feedback Loops I

- Closing the loop of shearing and Reynolds work



- Spectral ‘Predator-Prey’ equations



Prey → Drift waves, $\langle N \rangle$

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k \langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

Feedback Loops II

- DW-ZF turbulence 'nominally' described by predator-prey

growth suppression self-NL

$$\frac{\partial}{\partial t} N = \gamma N - \alpha V^2 N - \Delta\omega N^2,$$

$$\frac{\partial}{\partial t} V^2 = \alpha N V^2 - \gamma_{\mathrm{d}} V^2 - \gamma_{\mathrm{NL}}(V^2) V^2.$$

stress drive ZF damping NL ZF damping

Prey ≡ DW's (N) ↔ forward enstrophy scattering (induced diffusion to high k_r)

Predator ≡ ZF's (V^2) ↔ inverse energy scattering

Configuration ⇒ coupling coeffs.

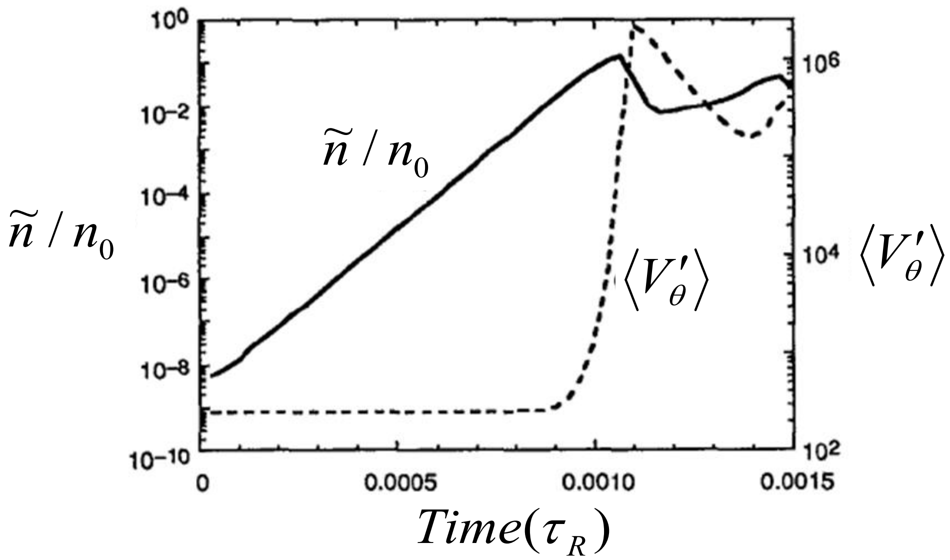
- Can have:
  - Fixed point $(\gamma / \Delta\omega);\quad (\gamma_d / \alpha, [(\gamma - \Delta\omega\gamma_d / \alpha)/\alpha]^{1/2})$
  - Limit cycle states,
  - depends on ratios of V dampings ⇒ phase lag

N.B. Suppression + Reynolds terms $\alpha V^2 N$ cancel for TOTAL momentum, energy

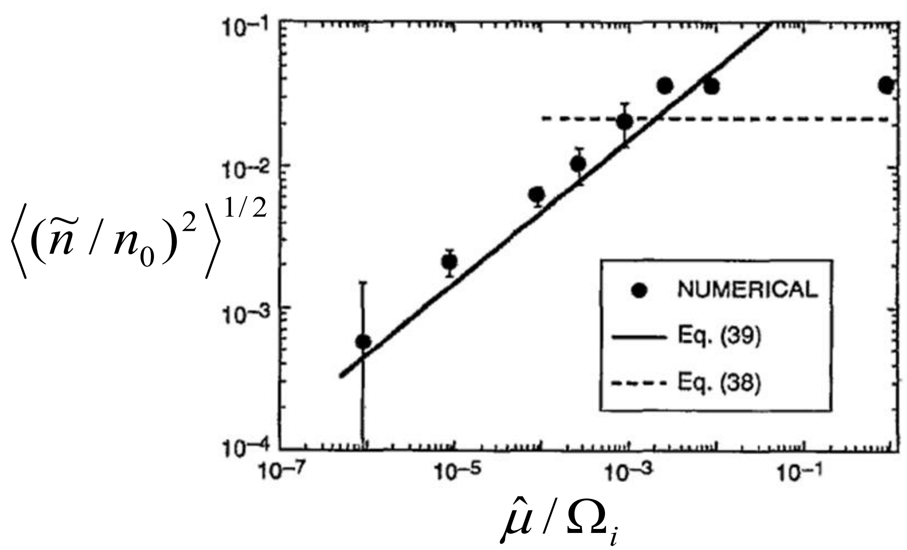
- Major concerns/omissions
  - Mean ExB coupling?
  - Turbulence drive γ ⇒ flux drive ⇔ avalanching? ⇒ not a local process
  - 1D ⇒ spatio-temporal problem (fronts, NL waves) ? ⇒ barrier width
  - NL flow damping ?

Feedback Loops III

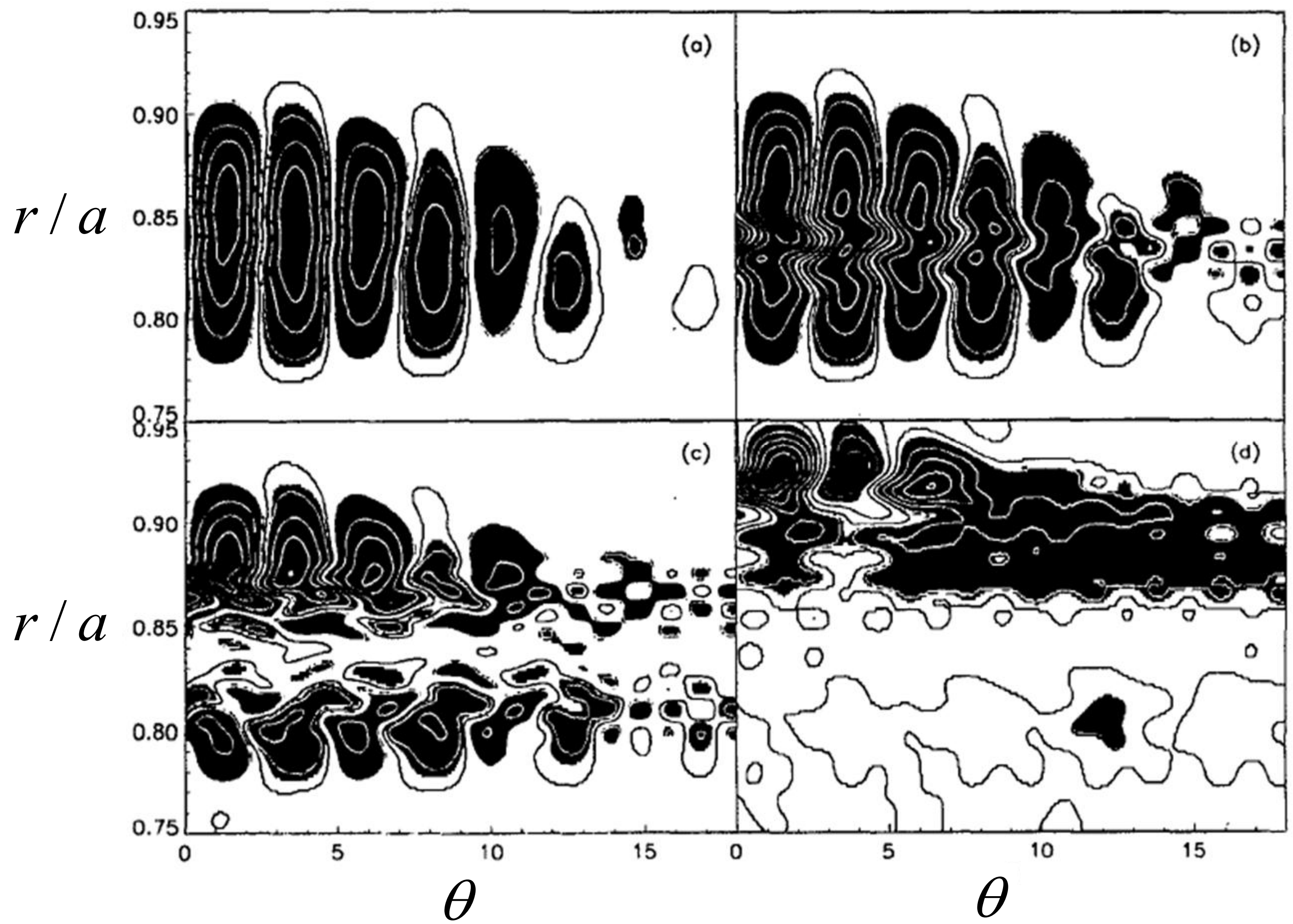
- Early simple simulations confirmed several aspects of modulational predator-prey dynamics (L. Charlton et. al. '94)



Shear flow grows above critical point



'With Flow' and 'No Flow'.
Scalings of $\langle (\tilde{n}/n_0)^2 \rangle$ appear. Role of damping evident



Generic picture of fluctuation scale reduction with flow shear

B) A Look Ahead

Current Applications to Selected Problems of Interest

Progress

- i) Zonal Flows with RMP
- ii) β-plane MHD and the Solar Tachocline

Provocation

- i) The PV and ExB Staircase
- ii) Zonal flows and spreading: What is the Interaction?

Pinnacle

Dynamics of the L→H Transition

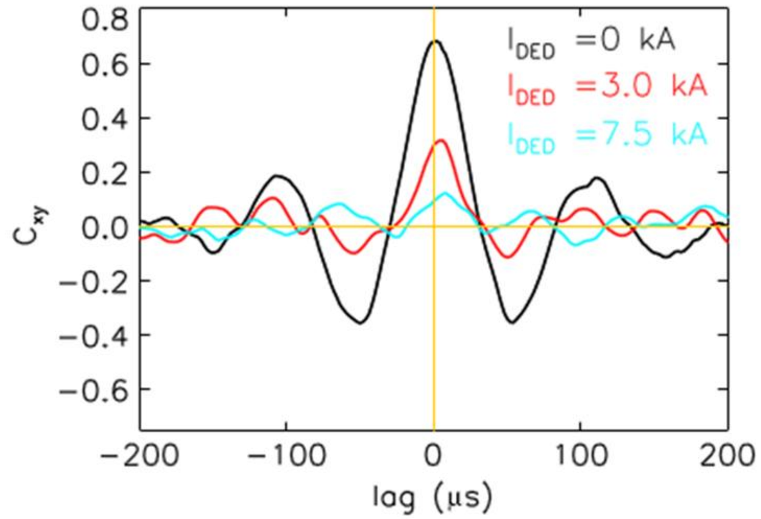
"What bifurcations, made by funksters, like mushrooms sprout both far and wide"

— Vladimir Sorokin, in "Day of the Oprichnik"

Progress I: ZF's with RMP (with M. Leconte)

- ITER 'crisis du jour': ELM Mitigation and Control
- Popular approach: RMP
- ? Impact on Confinement?

Y. Xu '11



⇒ RMP causes drop in fluctuation LRC, suggesting reduced Z.F. shearing

⇒ What is "cost-benefit ratio" of RMP?

- Physics:
  - in simple H-W model, polarization charge in zonal annulus evolves according:

$$\frac{dQ}{dt} = -\int dA \left[\left\langle \tilde{v}_x \tilde{\rho}_{pol} \right\rangle + \left(\frac{\delta B_r}{B_0} \right)^2 D_\parallel \frac{\partial}{\partial x} (\langle \phi \rangle - \langle n \rangle) \right] \Bigg|_{r_1}^{r_2}$$

$$\langle B_r J_\parallel \rangle = \langle B_r^2 \rangle \langle J_r \rangle + \langle \widetilde{B}_r \widetilde{J}_\parallel \rangle$$

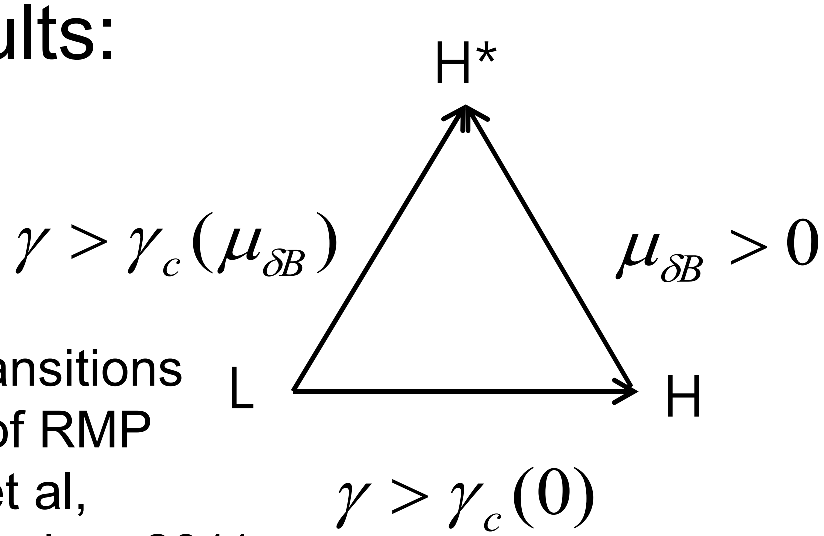
fluctuations

radial current along tilted, static lines

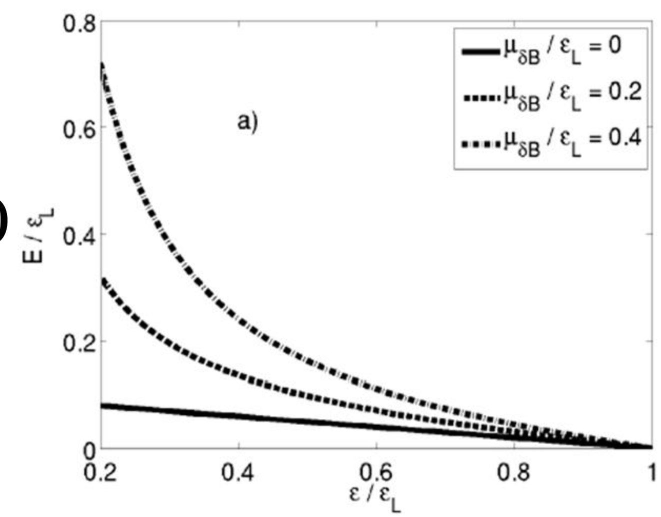
  - Key point: δB_r of RMP induces radial electron current → enters charge balance

Progress I, cont'd

- Implications
  - δB_r linearly couples zonal $\hat{\phi}$ and zonal $\hat{n}$
  - Weak RMP → correction, strong RMP → $\langle E_r \rangle_{ZF} \cong -T_e \partial_r \langle n \rangle / |e|$
- Equations: $$\frac{d}{dt}\delta n_q + D_T q^2 \delta n_q + i b_q(\delta\phi_q - (1-c)\delta n_q) - D_{RMP} q^2 (\delta\phi_q - \delta n_q) = 0$$
$$\frac{d}{dt}\delta\phi_q + \mu\delta\phi_q - a_q(\delta\phi_q - (1-c)\delta n_q) + \frac{D_{RMP}}{\rho_s^2}(\delta\phi_q - \delta n_q) = 0$$
- Results:



P_{thres} ↑ for transitions in presence of RMP c.f. F. Ryter et al, H-mode Workshop 2011



$E_{ZF}/\mathcal{E}_L$ vs $\mathcal{E}/\mathcal{E}_L$ for various RMP coupling strengths

Progress II : β-plane MHD (with S.M. Tobias, D.W. Hughes)

Model

- Thin layer of shallow magneto fluid, i.e. solar tachocline
- β-plane MHD ~ 2D MHD + β-offset   i.e. solar tachocline

$$\partial_t \nabla^2 \phi + \nabla \phi \times \hat{z} \cdot \nabla \nabla^2 \phi - \nu \nabla^2 \nabla^2 \phi = \beta \partial_x \phi + B_0 \partial_x \nabla^2 A + \nabla A \times \hat{z} \cdot \nabla \nabla^2 A + \tilde{f}$$

$$\partial_t A + \nabla \phi \times \hat{z} \cdot \nabla A = B_0 \partial_x \phi + \eta \nabla^2 A \qquad \vec{B}_0 = B_0 \hat{x}$$

- Linear waves: Rossby – Alfven $\omega^2 + \omega\beta \frac{k_x}{k^2} - k_x^2 V_A^2 = 0$ (R. Hide)
- cf P.D., et al; Tachocline volume, CUP (2007)

  S. Tobias, et al: ApJ (2007)

Progress II, cont'd

Observation re: What happens?

- Turbulence → stretch field → $\langle \tilde{B}^2 \rangle >> B_0^2$ i.e. $\langle \tilde{B}^2 \rangle / B_0^2 \sim R_m$
  (ala Zeldovich)
- Cascades : - forward or inverse?
  - MHD or Rossby dynamics dominant !?
- PV transport: $\frac{dQ}{dt} = -\int dA \langle \tilde{v}\tilde{q} \rangle$ ⟶ net change in charge content due PV/polarization charge flux

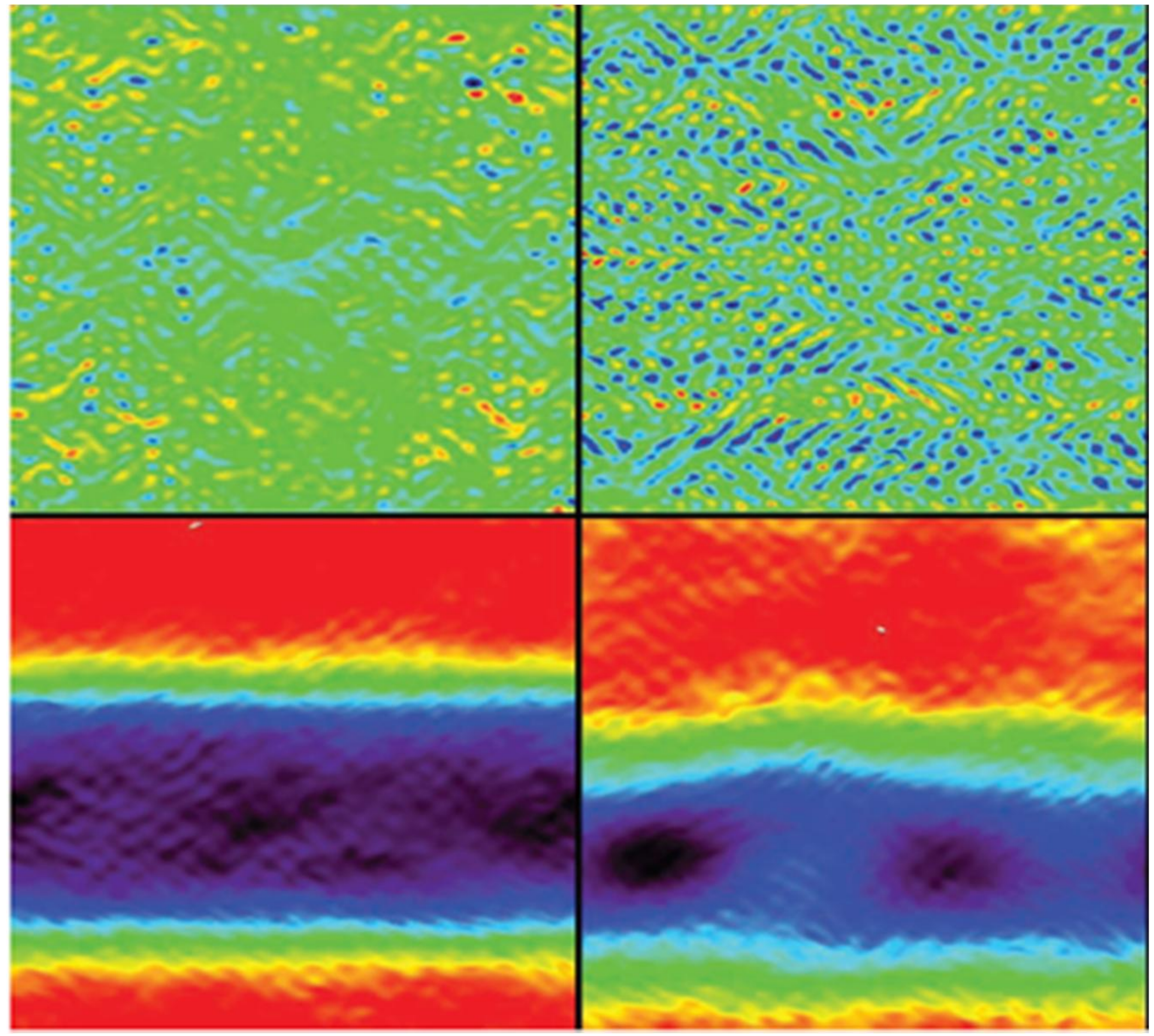
Now $\frac{dQ}{dt} = -\int dA \left[\langle \tilde{v}\tilde{q} \rangle - \langle \tilde{B}_r \tilde{J}_{\parallel} \rangle \right] = -\int dA \partial_x \left\{ \langle \tilde{v}_x \tilde{v}_y \rangle - \langle \tilde{B}_x \tilde{B}_y \rangle \right\}$ ⟶ Reynolds mis-match

PV flux   current along tilted lines   ⟶ vanishes for Alfvenized state

Taylor: $\langle \tilde{B}_x \tilde{J}_{\parallel} \rangle = -\partial_x \langle \tilde{B}_x \tilde{B}_y \rangle$

Progress II, cont'd

- **With Field**



$B_0 = 10^{-1}$

$B_0 = 10^{-2}$

$B_0 = 0$

$B_0 = 10^{-3}$

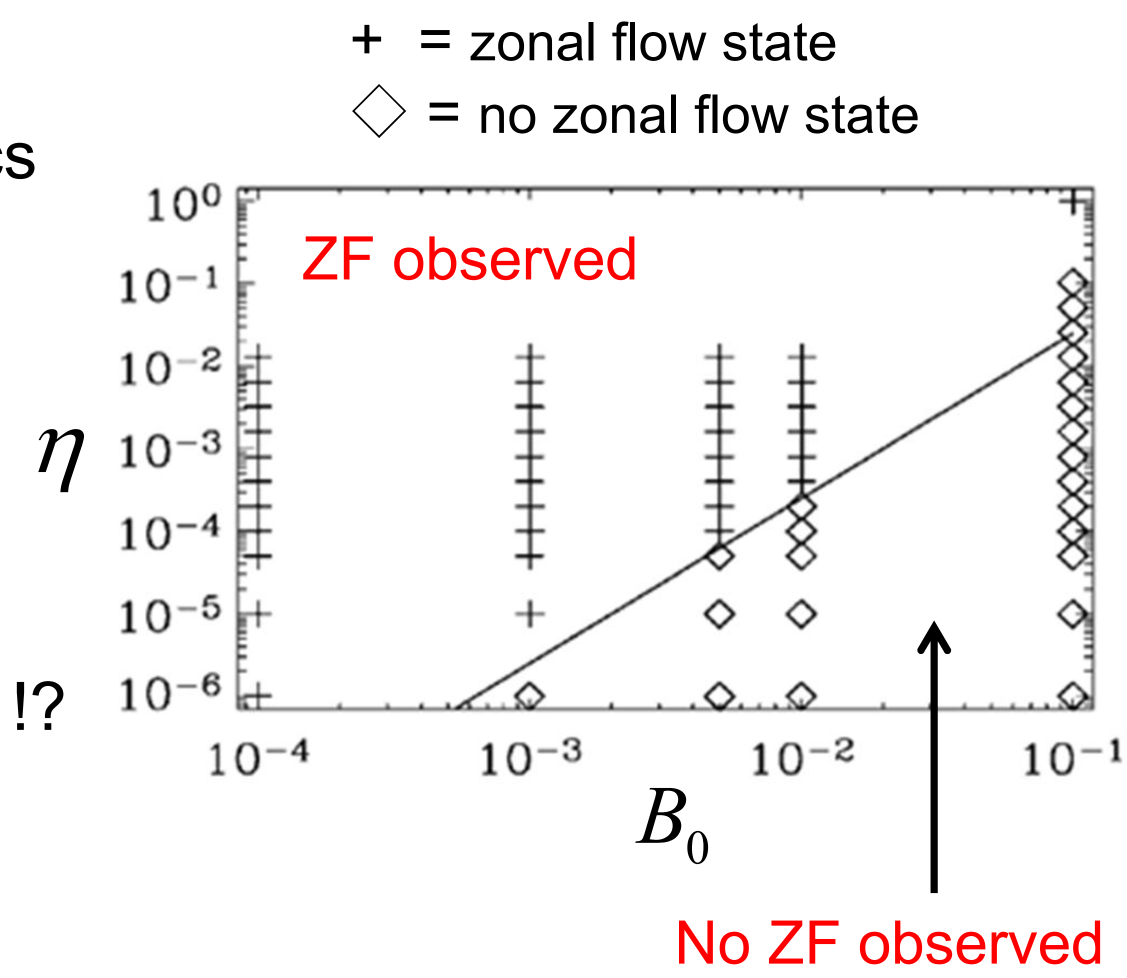
Progress II, cont'd

- Control Parameters for $\vec{\tilde{B}}$ enter Z.F. dynamics

  Like RMP, Ohm's law regulates Z.F.

- Recall
  - $\langle \tilde{v}^2 \rangle$ vs $\langle \tilde{B}^2 \rangle$
  - $\langle \tilde{B}^2 \rangle \sim B_0^2 R_m$ $\longrightarrow$ origin of B_0^2 / η scaling !?

- Further study $\rightarrow$ differentiate between :
  - cross phase in $\langle \tilde{v}_r \tilde{q} \rangle$ and O.R. vs J.C.M
  - orientation : $\vec{B} \parallel \vec{V}$ vs $\vec{B} \perp \vec{V}$
  - spectral evolution

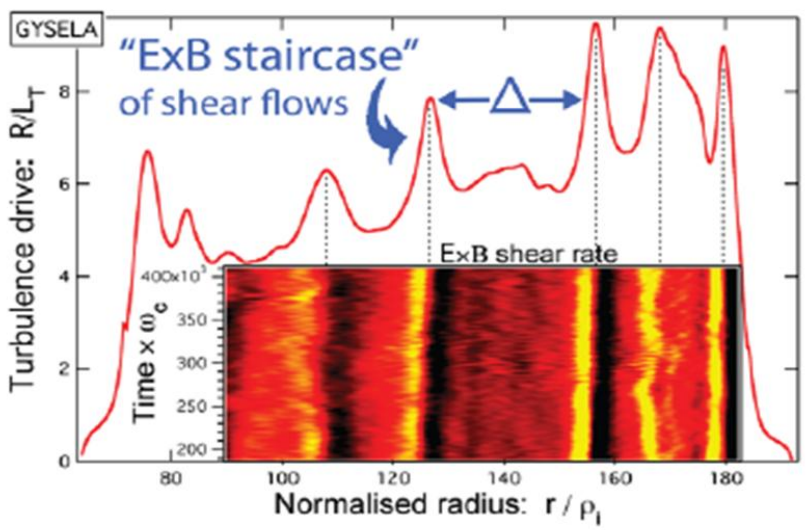


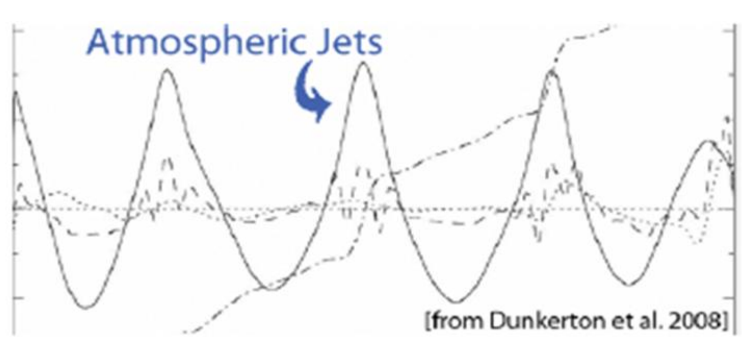
? How does system solve the pattern selection problem of Zonal Flow vs Avalanches?

Provocation I: Staircase and Nonlocality
(with G. Dif-Pradalier, et. al.)

UCSD

Analogy with geophysics: the '**E** × **B** *staircase*'





$Q = -n\chi(r)\nabla T$ ➠ $\boxed{Q = -\int \kappa(r, r')\nabla T(r')\,dr'}$

- '**E** × **B** staircase' width ≡ kernel width Δ
- coherent, persistent, jet-like pattern
  ➠ the '**E** × **B** staircase'
- staircase NOT related to low order rationals!

Dif-Pradalier, Phys Rev E. 2010

Guilhem Dif-Pradalier APS–DPP meeting, Atlanta, Nov. 2009

Avalanches ↔ 'Non-locality'

- Non-locality?

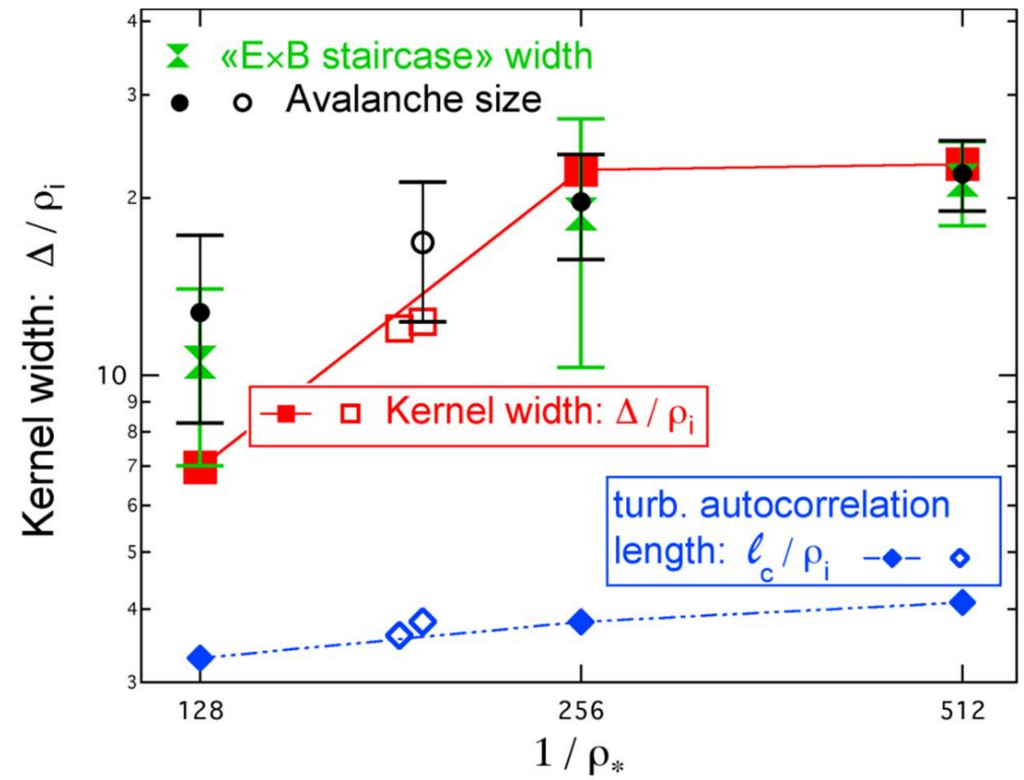
$$Q = -\chi \nabla T \qquad \Rightarrow \qquad Q = -\int dr' \int dt' K(r-r', t-t') \nabla T(r', t')$$

$\Delta r_K >> \Delta r_{cor} \quad \tau_K >> \tau_{cor}$

G. Dif-Pradalier 2010

$$Q = -\int dr' K(r-r') \nabla T(r')$$

$$K \cong s^2 / [(r-r')^2 + \Delta_{aval}^2]$$



– Non-locality and/or Nonlinearity $(\partial_t I - \partial_r \chi I \partial_r I = \gamma I - \alpha I^2)$

Physics? Local but fast

Provocation I, cont'd

- The point:
  - fit: $Q = -\int dr' \kappa(r,r') \nabla T(r')$ $\quad \kappa(r,r') \sim \dfrac{S^2}{(r-r')^2 + \Delta^2}$ $\rightarrow$ some range in exponent
  - $\Delta >> \Delta_c$ i.e. $\Delta \sim$ Avalanche scale >> $\Delta_c \sim$ correlation scale
  - Staircase 'steps' separated by Δ ! $\rightarrow$ stochastic avalanches produce quasi-regular flow pattern!?

    N.B.

    - The notion of a staircase is not new – especially in systems with natural periodicity (i.e. NL wave breaking…)
    - What IS new is the connection to stochastic avalanches, independent of geometry
  - What is process of self-organization linking avalanche scale to zonal pattern step?

    i.e. How extend predator-prey feedback model to encompass both avalanche and zonal flow staircase? Self-consistency is crucial!

- The Lessons:
  - Mesoscale structure formation is a consequence of multiple feedbacks in space and time
  - System can solve the pattern selection problem of ZF’s vs avalanches by spatial decomposition into ‘staircase steps’ and avalanche zones
  - Feedback loop physics of paramount importance!

C) Pinnacle

Z.F.'s and the Dynamics of the L→H Transition

- L→H transition (F. Wagner '82) has driven considerable research on shear flows
- Tremendous progress in recent experiments:

  G. Conway, T. Estrada and C. Hidalgo,

  L. Schmitz, G. McKee and Z. Yan,

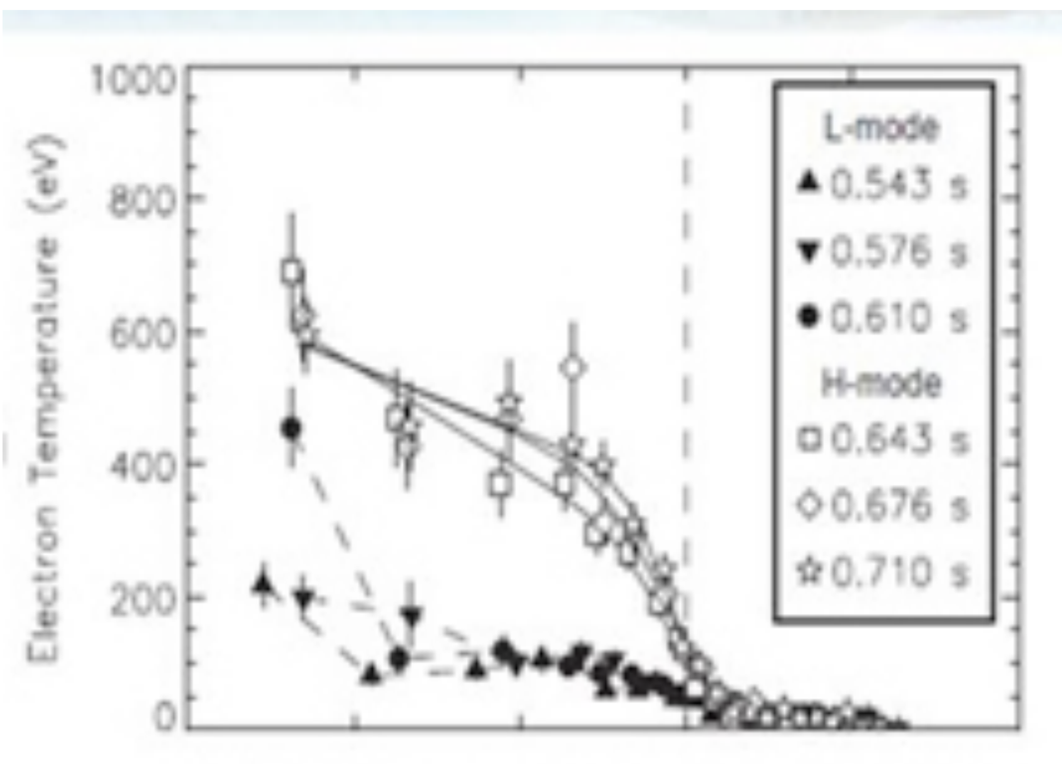
  K. Kamiya and K. Ida, G.S. Xu,

  A. Hubbard, S. J. Zweben

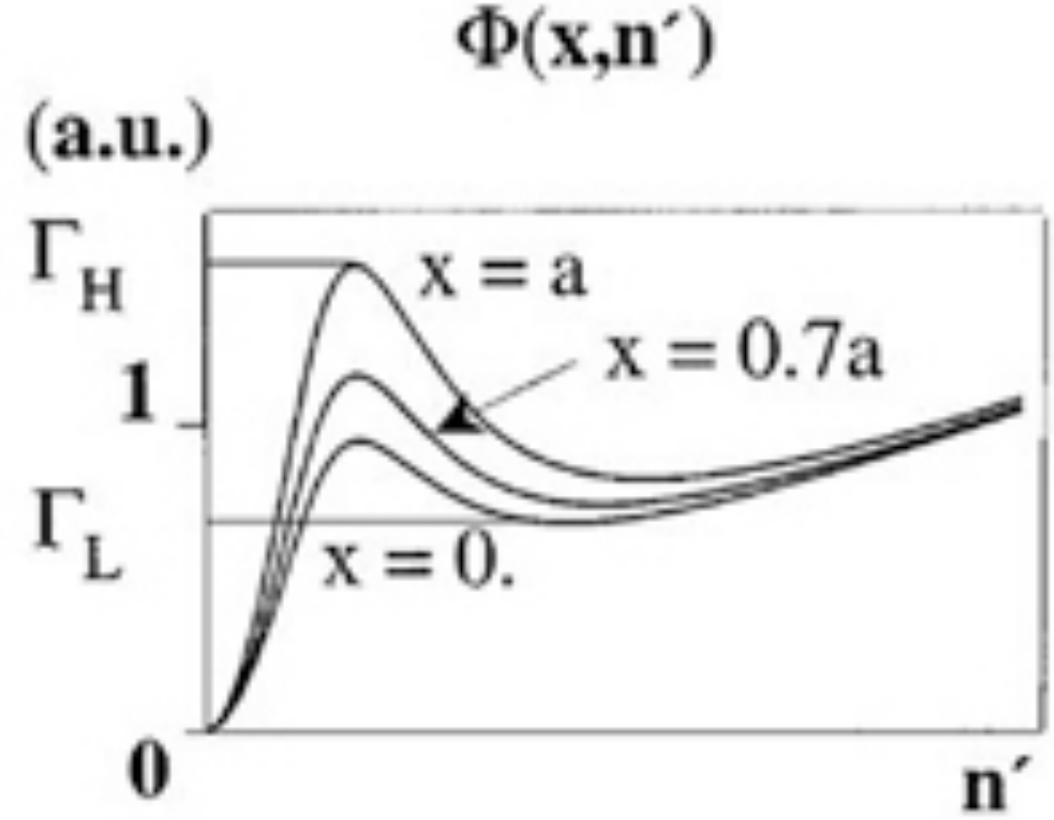
- Seems like we are almost there …

BUT: "It ain't over till its over" – an eastern (division) Yogi

What is the H-mode? What is a transport barrier?



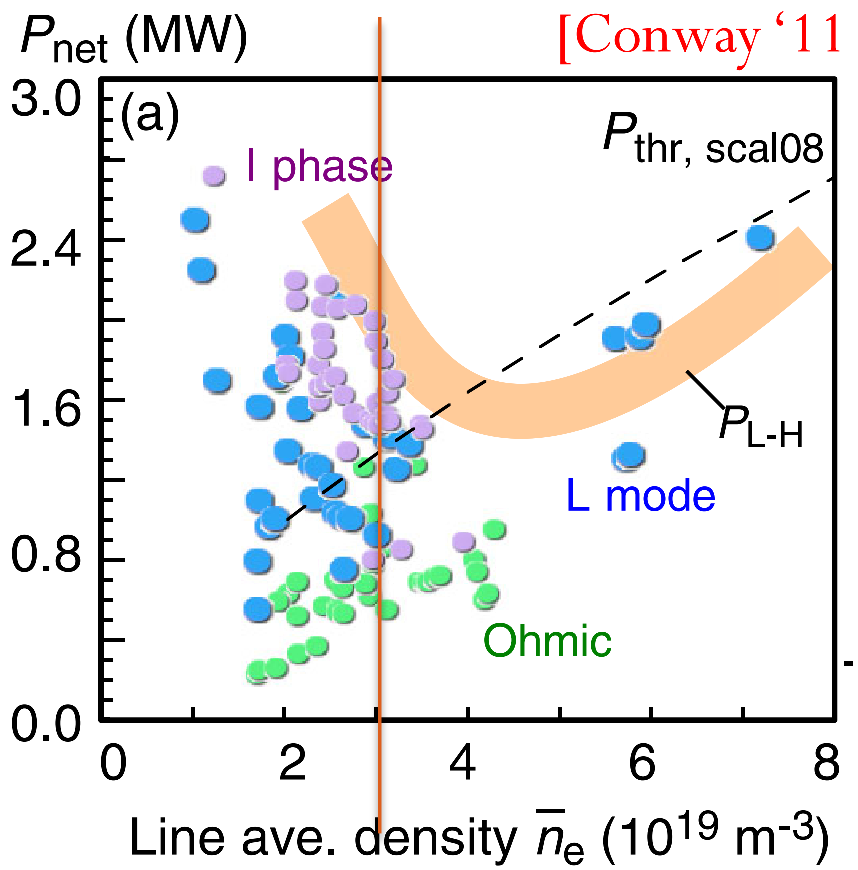
J.W. Huges et al., PSFC/JA-05-35

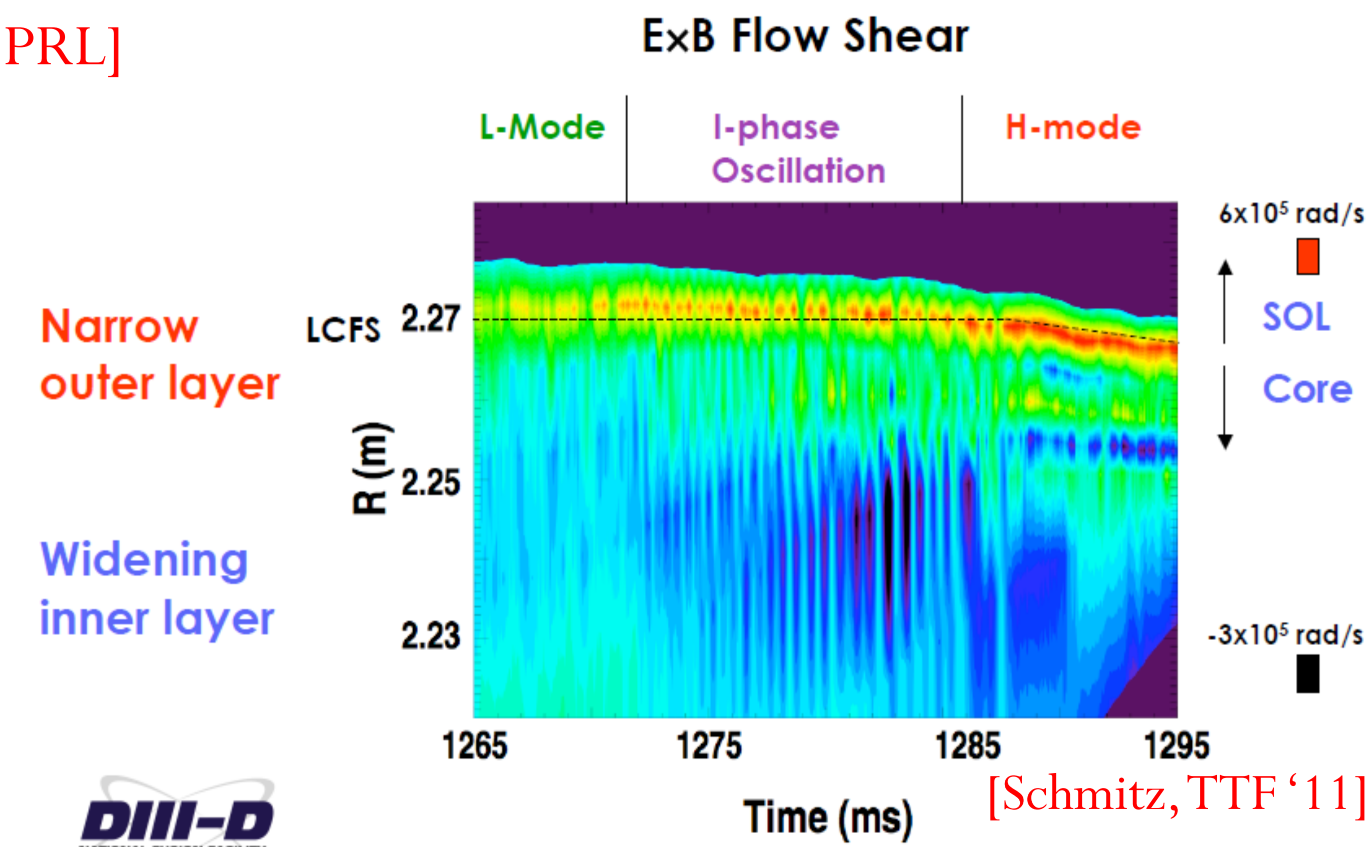


Lebedev et al., Phys. Plasmas 1997

Experimental motivation

[Conway '11 PRL]





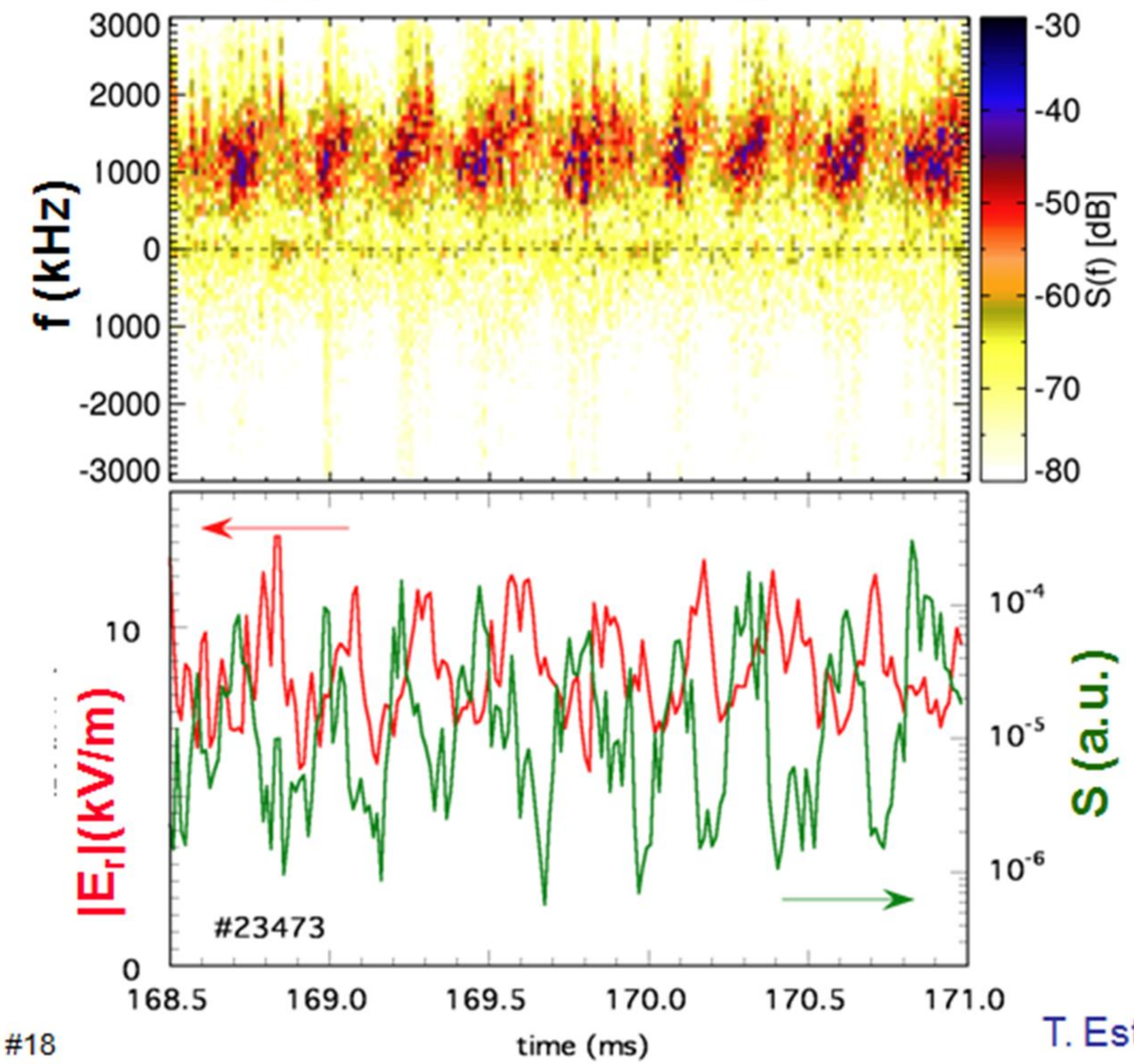
[Schmitz, TTF '11]

- L-H threshold Power in low density region (typically lower than $3\times10^{19}m^{-3}$)
- I-phase as a transient phase between low and high confinement, i.e. L→I→H transition.
- Limit cycle oscillation in prior to the transition in TJ-II[Estrada '10 EPL], NSTX[Zweben '10 PoP], ASDEX Upgrade[Conway '11 PRL], EAST[Xu '11 PRL]
- Radial structure of mean flow shear in the I-phase limit-cycle oscillation
  - Dual shear layer in DIII-D [Schmitz, TTF '11]
- Poloidal rotation involving in the transition process in JT-60U [Kamiya '10 PRL]

Flows and turbulence dynamics

Laboratorio Nacional Fusión

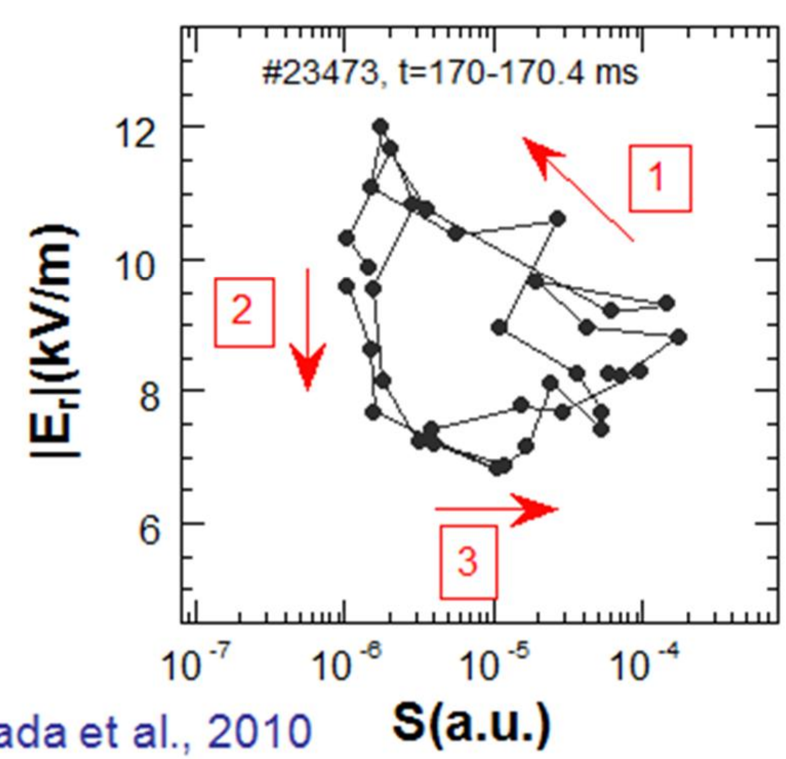
Doppler reflectometry



#18

The time evolution shows a predator-prey behaviour:

Periodic evolution of E_r and ñ with the E_r following ñ with a phase delay of 90º.



T. Estrada et al., 2010

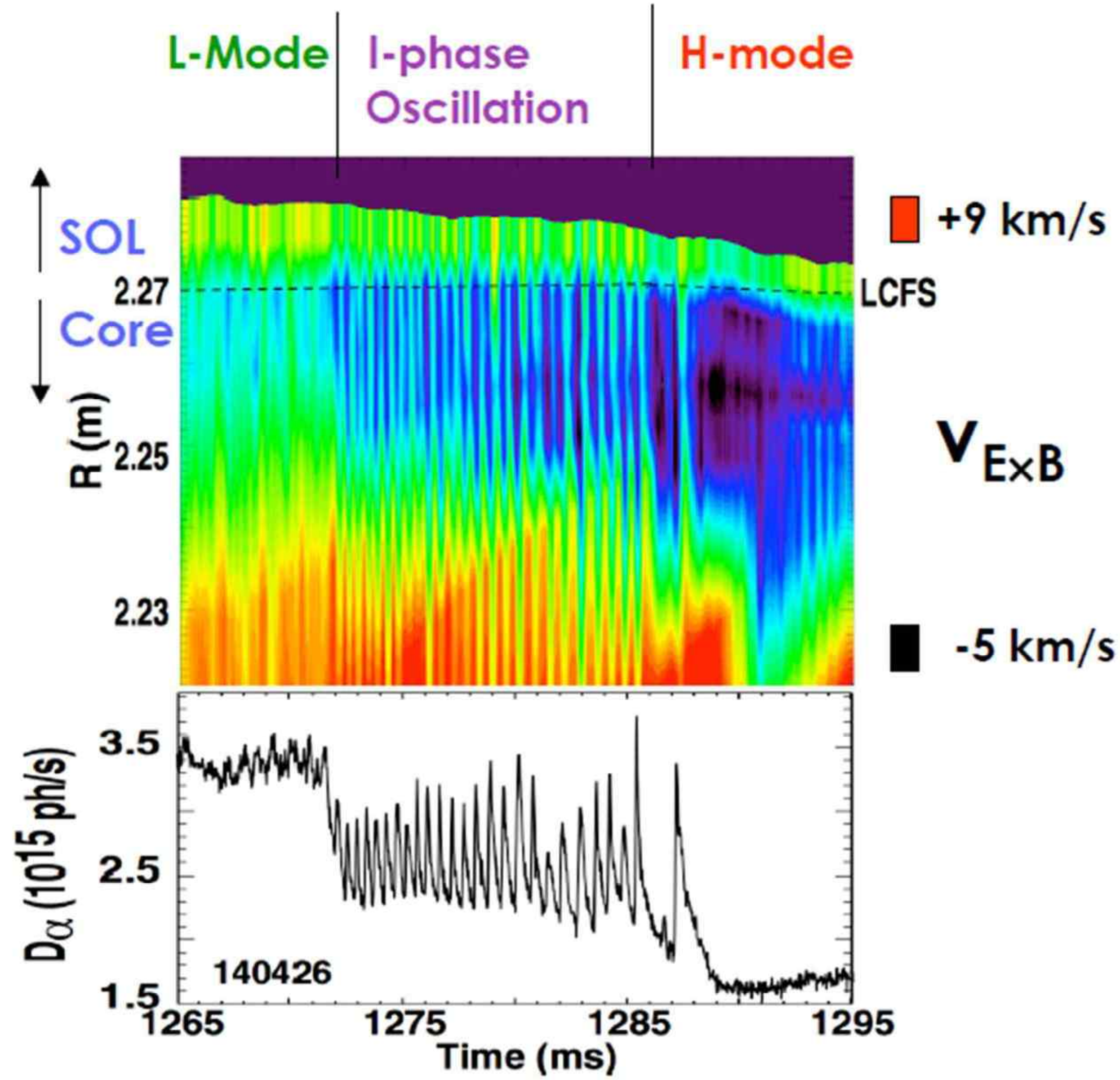
s

The Oscillating Flow Layer Widens Radially (Frequency Decreases) - Steady Flow after Final H-Mode Transition

A weak E×B flow layer exists in L-mode (L-mode shear layer)

At the I-phase transition, the E×B flow becomes more negative first near the separatrix, flow layer then propagates inward

The flow becomes steady at the final H-mode transition (after one final transient)



L. Schmitz TTF'11

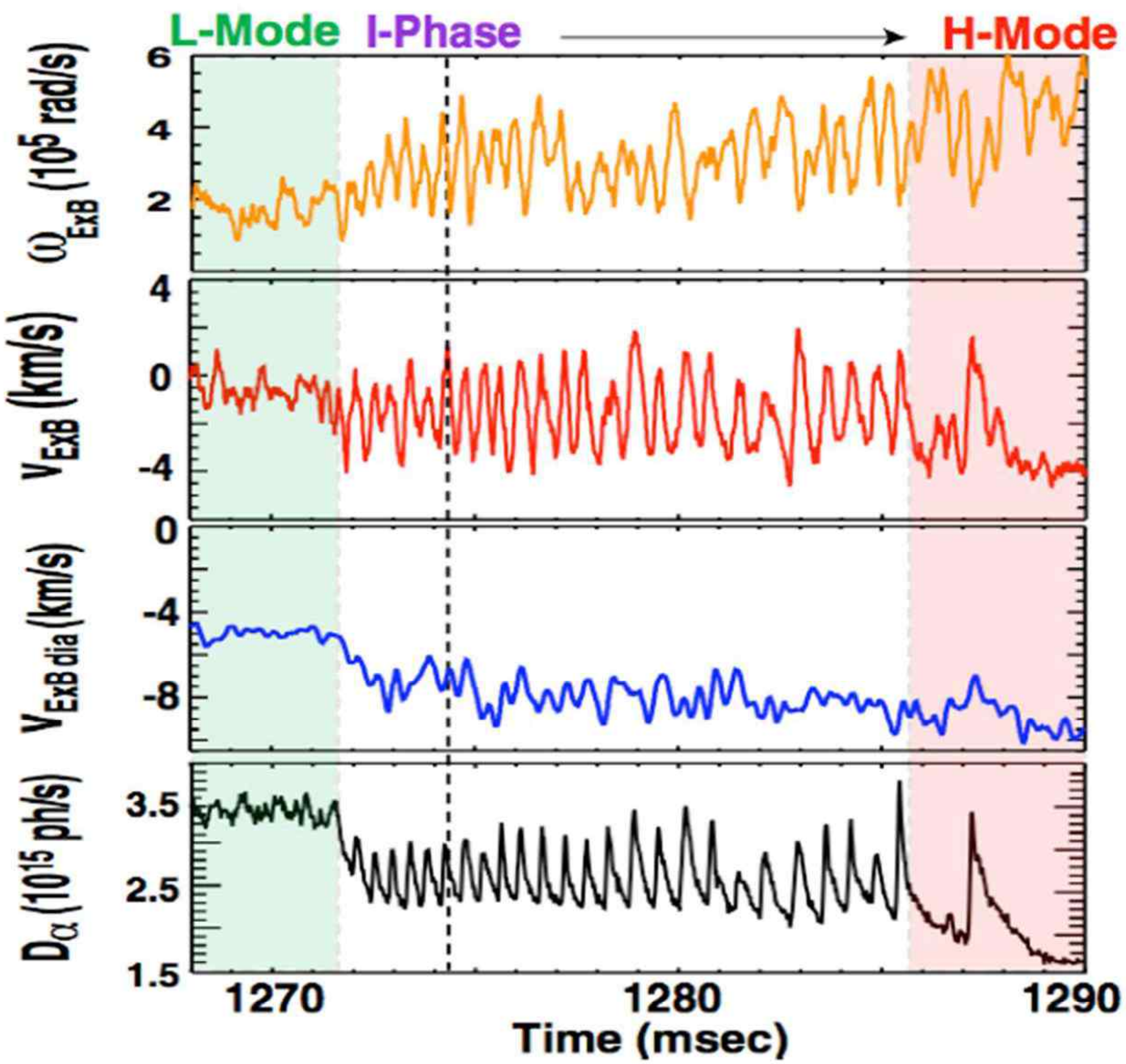
During the I-phase, the Mean Shear $\langle \omega_{ExB} \rangle$ Increases with Time and Eventually Dominates

Outer layer Shearing Rate (Mean flow+ ZF)

ExB Flow from DBS (includes ZF)

Diamagnetic component of ExB flow (from ion pressure Profile)

R~2.265m



DIII-D NATIONAL FUSION FACILITY SAN DIEGO

L. Schmitz TTF'11

→ EAST Results (G.S. Xu, et.al., ’11)

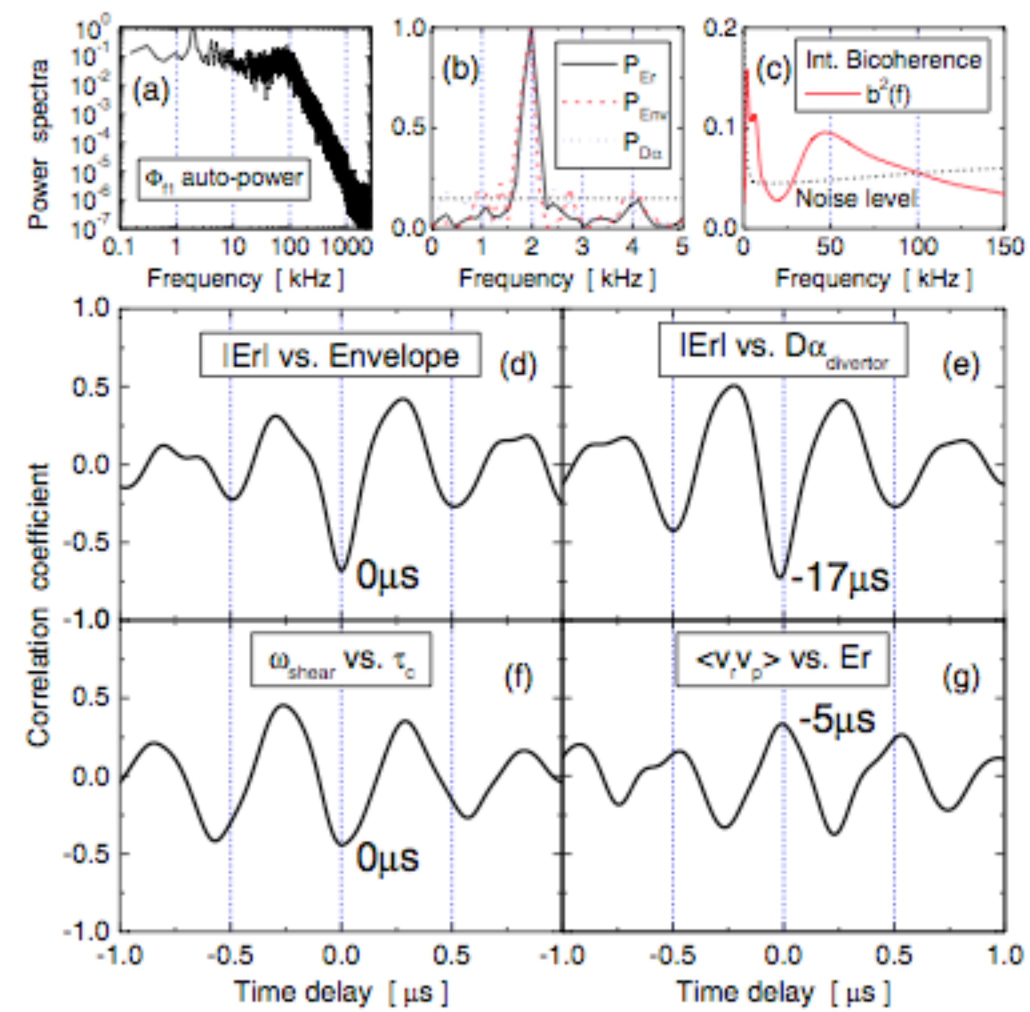
→ hardened, reciprocating probes

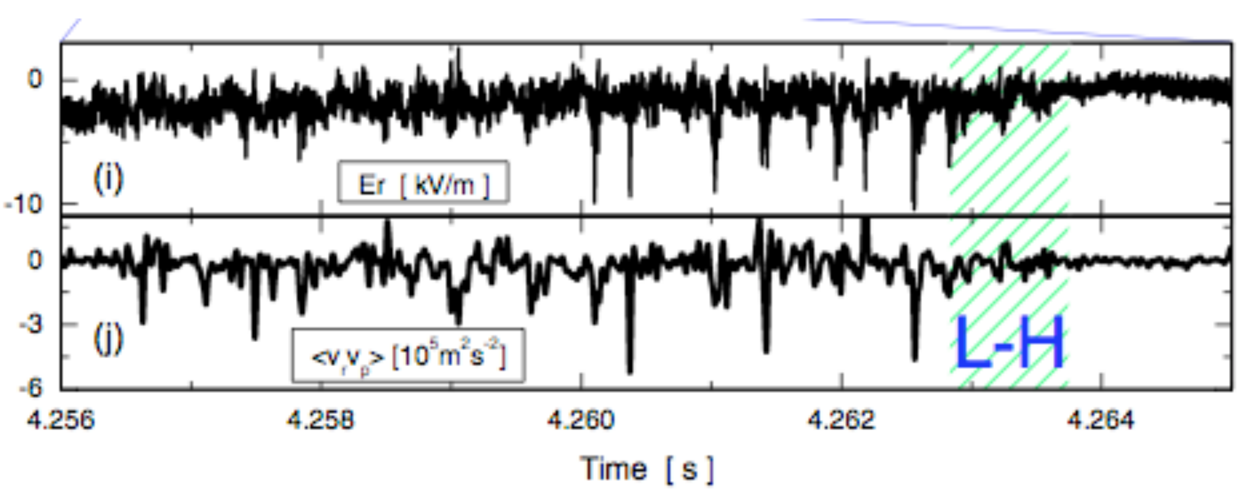
→ quasi-periodic Er oscillation (f < 4 kHz), with associated turbulence modulation

→ E_r and $\langle \tilde{v}_r \tilde{v}_\theta \rangle_E$ exhibits correlated spikes prior to transition

→ support key role of zonal flow in L→H transition

Power spectra peaks at f < 4kHz





spike in E_r and $\langle \tilde{v}_r \tilde{v}_\theta \rangle_E$

Pinnacle, cont'd

- ∇P coupling

  - γ_L *drive*
  - $\langle V_E \rangle'$

  \+

  Z.F.

$$\partial_t \varepsilon = \varepsilon N - a_1 \varepsilon^2 - a_2 V^2 \varepsilon - a_3 V_{ZF}^2 \varepsilon$$

$$\partial_t V_{ZF} = b_1 \frac{\varepsilon V_{ZF}}{1 + b_2 V^2} - b_3 V_{ZF}$$

$$\partial_t N = -c_1 \varepsilon N - c_2 N + \langle Q \rangle + \tilde{Q}$$

$\mathcal{E}$ ≡ DW energy

V_{ZF} ≡ ZF shear

N ≡ $\nabla\langle P \rangle$ ≡ pressure gradient

V = dN^2 (radial force balance)

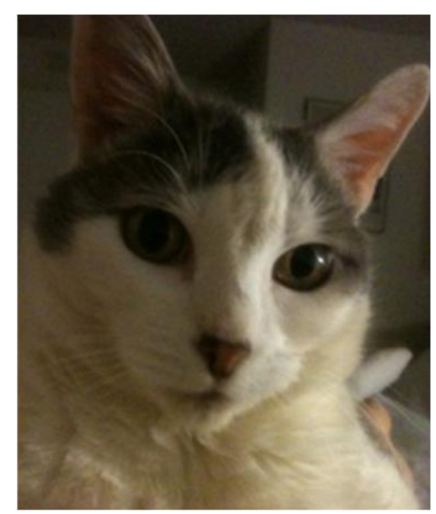
- Simplest example of 2 predator + 1 prey problem (E. Kim, P.D., 2003 see also: Malkov, P.D., 2009)

  i.e. prey sustains predators
  predators limit prey } usual feedback

  now:
  - 2 predators (*ZF*, $\nabla\langle P \rangle$) compete
  - $\nabla\langle P \rangle$ as both drive and predator
  - avalanches → multiplicative noise

- Relevance: LH transition, ITB
  - Builds on insights from Itoh's, Hinton
  - ZF ⇒ triggers
  - $\nabla\langle P \rangle$ ⇒ 'locking in'

Multiple predators are possible



Feedback Loops III, cont'd

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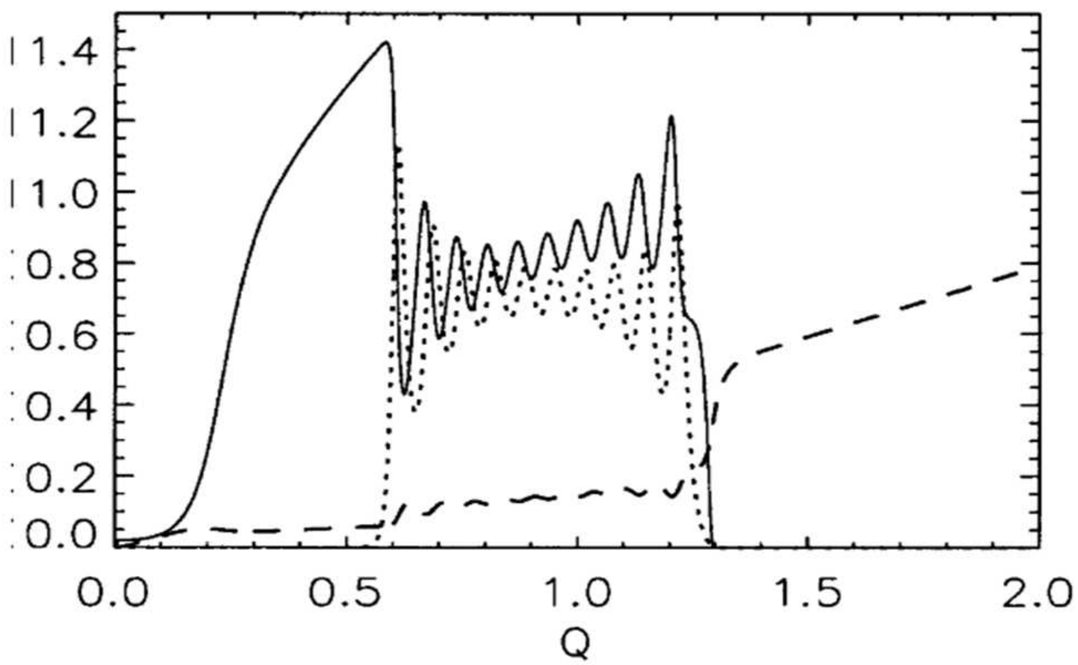


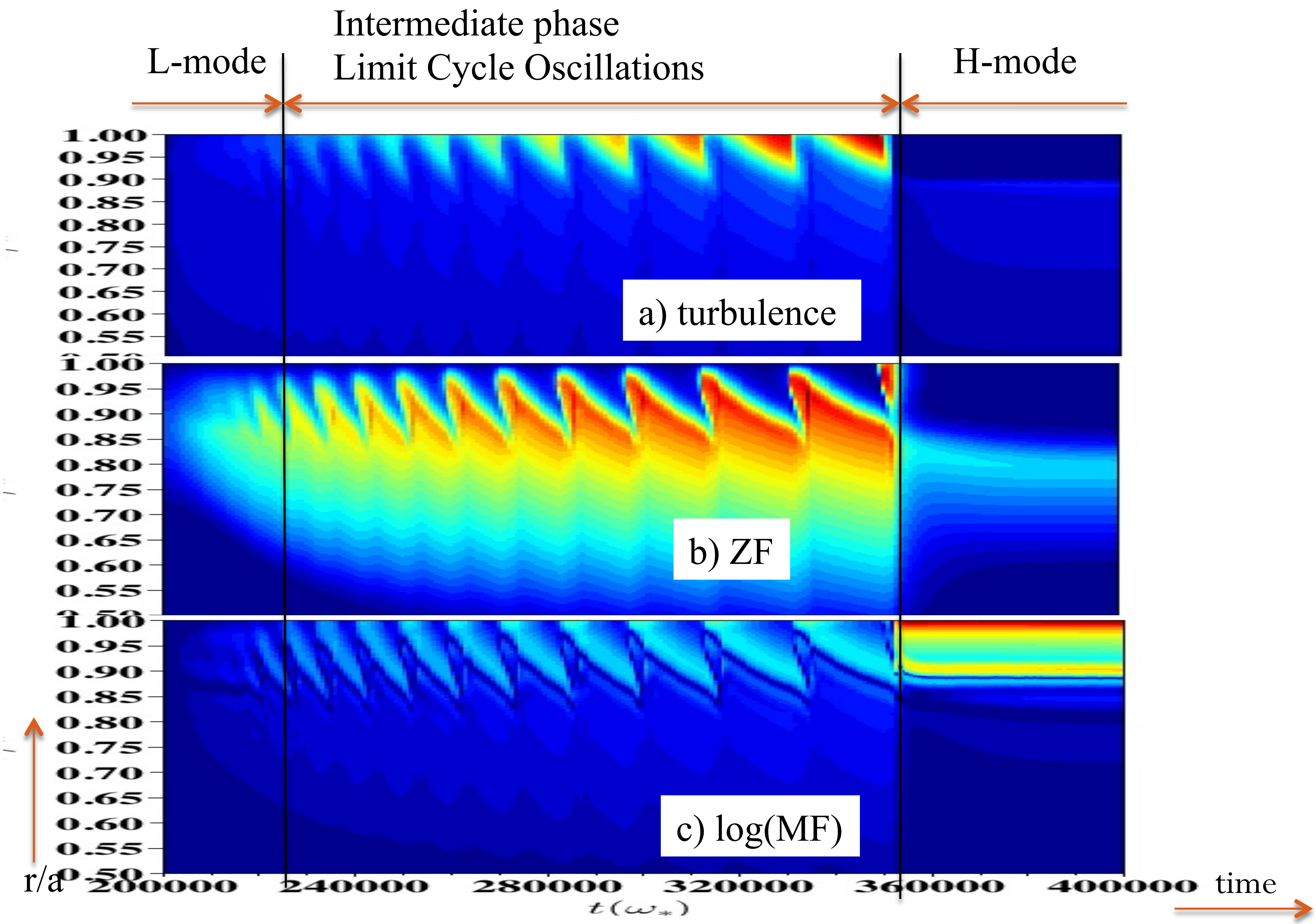
Solid - $\mathcal{E}$

Dotted - V_{ZF}

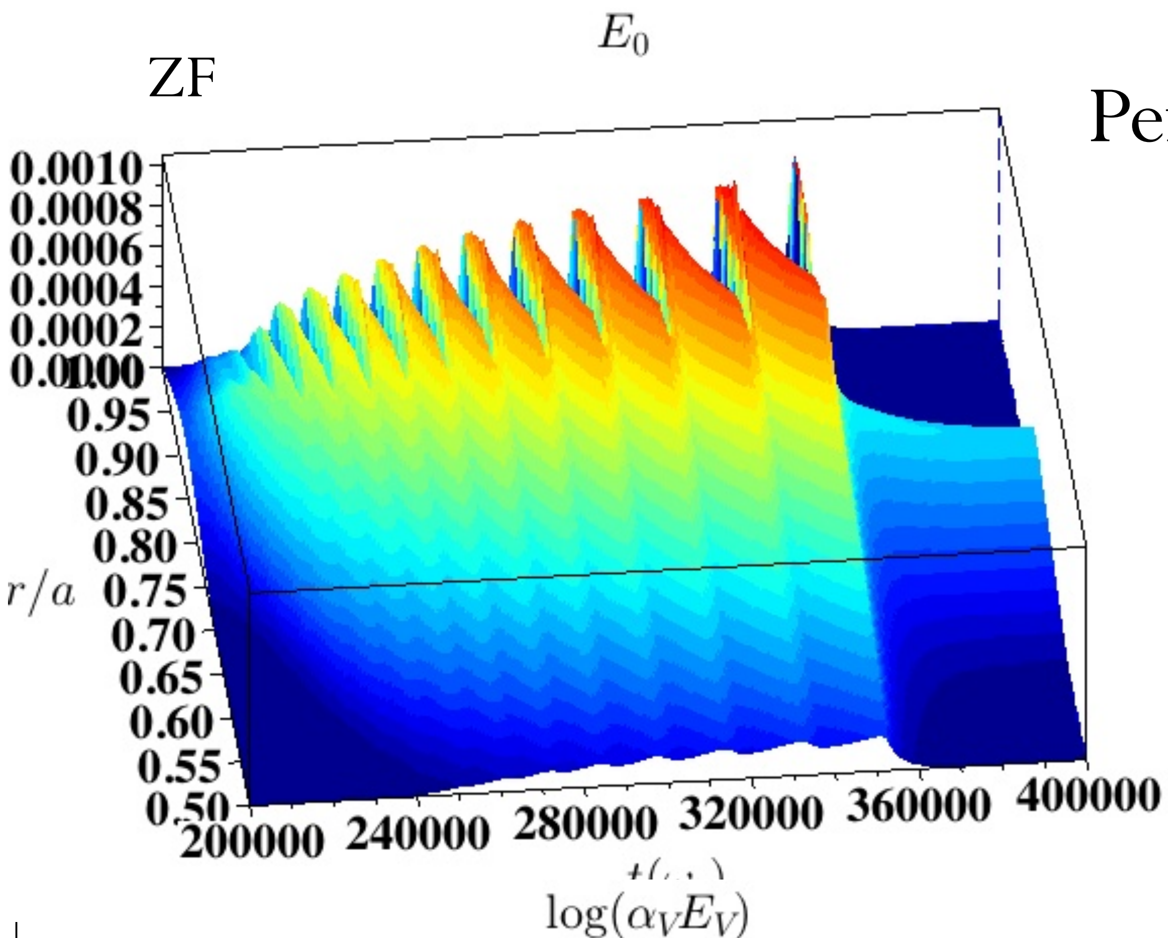
Dashed - $\nabla\langle P\rangle$

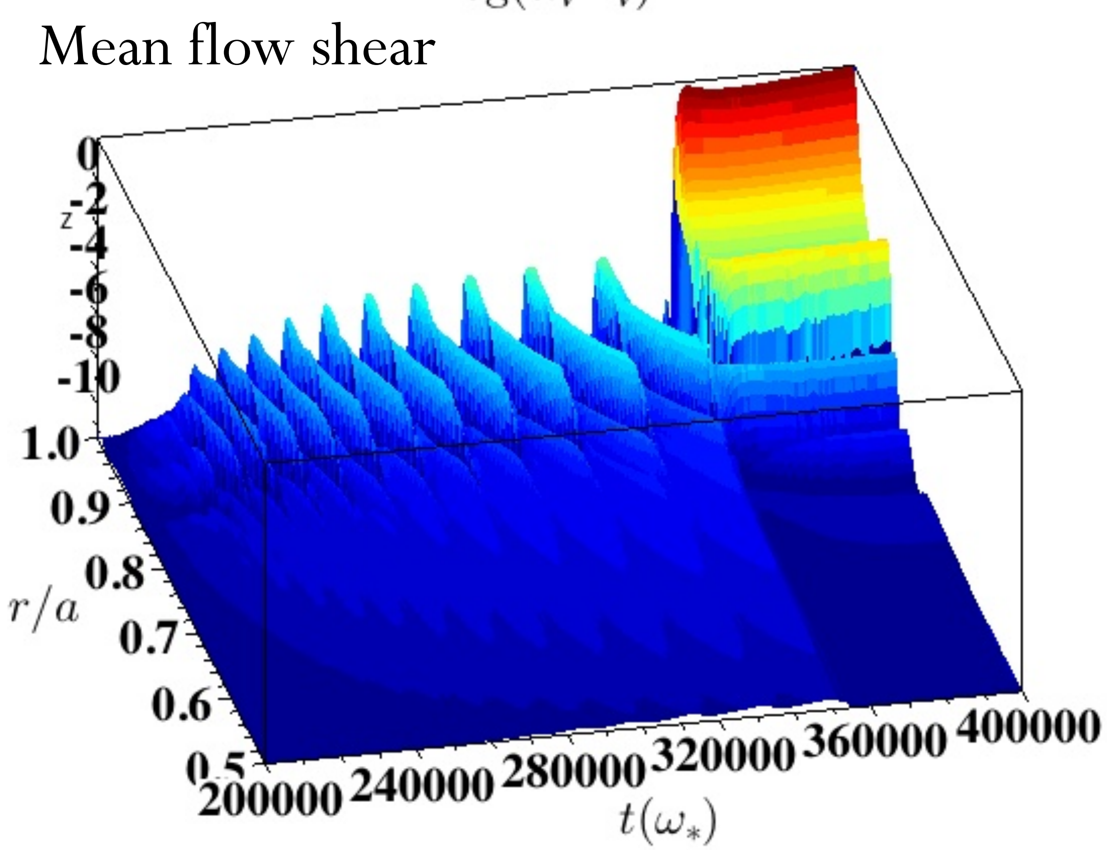
- Observations:
  - ZF's trigger transition, $\nabla\langle P\rangle$ and $\langle V\rangle$ lock it in
  - Period of dithering, pulsations .... during ZF, $\nabla\langle P\rangle$ oscillation as $Q\uparrow$
  - Phase between $\mathcal{E}$, V_{ZF} , $\nabla\langle P\rangle$ varies as Q increases
  - $\nabla\langle P\rangle \Leftrightarrow$ ZF interaction $\Rightarrow$ effect on wave form

Slow Power Ramp Indicates L→I→H Evolution.

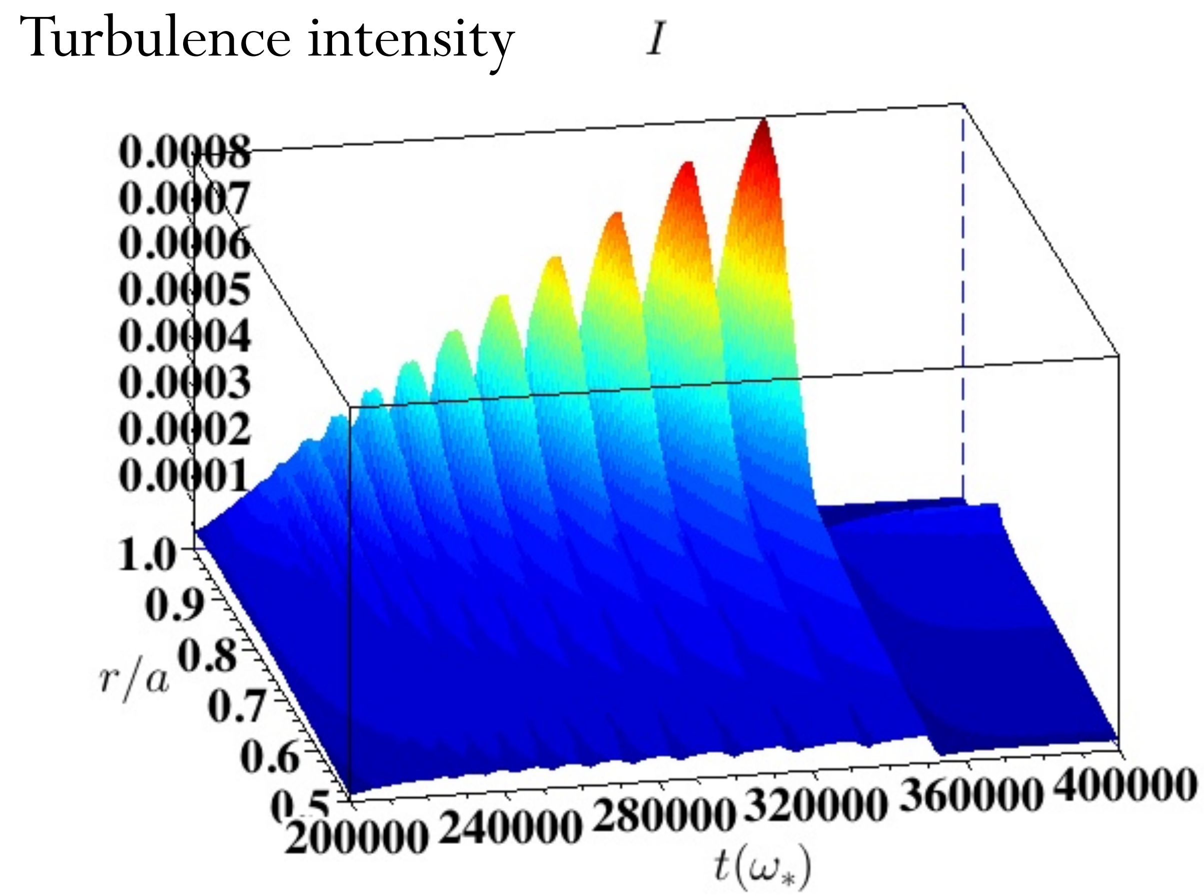


Cycle is propagating nonlinear wave in edge layer



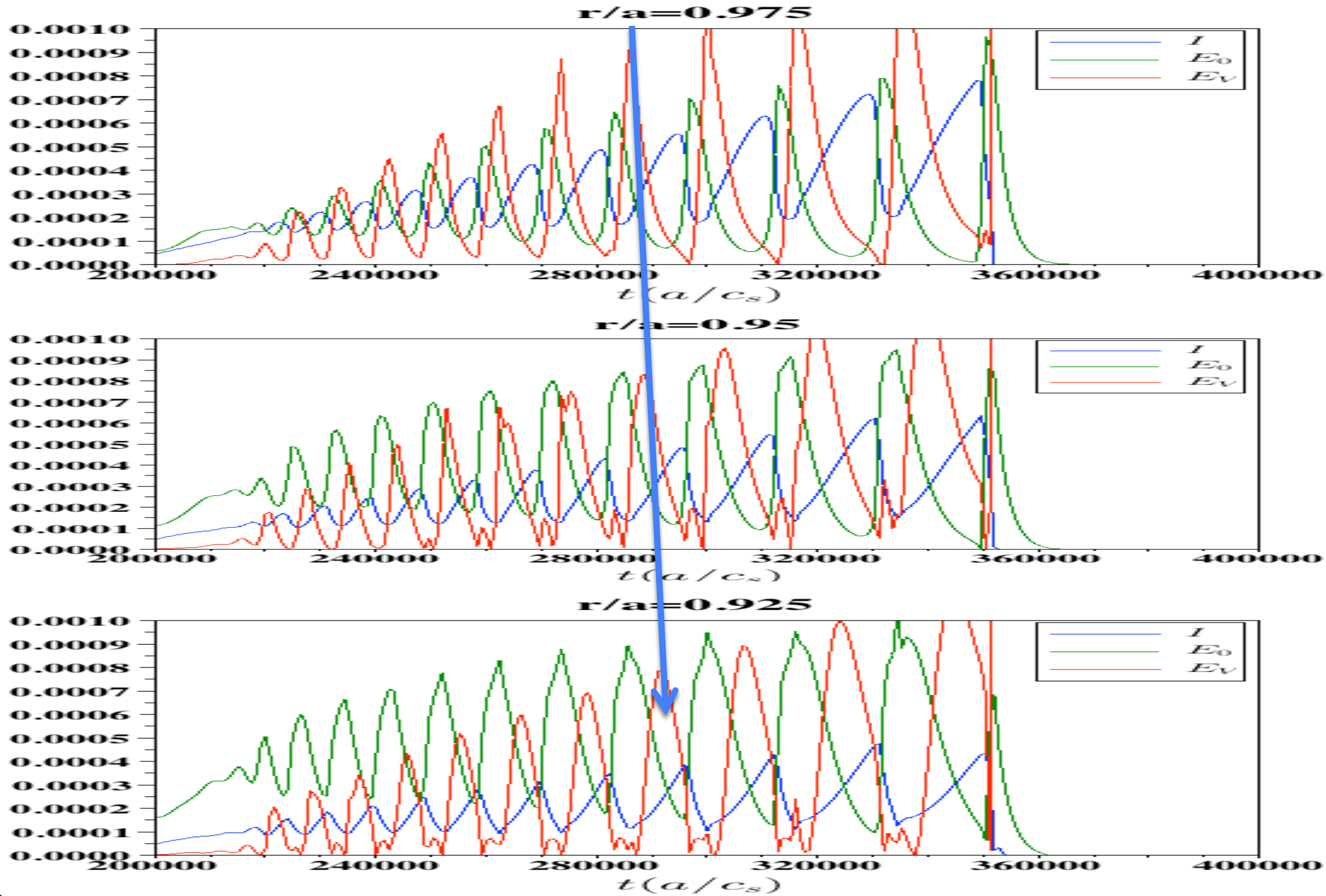


Period of cycle increases approaching transition.



- Turbulence intensity peaks just prior to transition.
- Mean shear (i.e. profiles) also oscillates in I-phase.

Mean shear location comparisons indicate inward propagation.



Pinnacle, cont'd

- For $P \sim P_{th}$, cyclic / dithering oscillations observed in flows, turbulence
- Multi-shear flow competition at work in transition process
- Flow structure evolves as transition progresses
- Many aspects of dynamics well described by multi-predator shearing models ala' K+D
- Variety of results, hints, suggestions, proclamations as to precise trigger… GAM, ZF, Mean ExB_0 Flow, Mean Poloidal Flow…

  Need there be a unique route to transition?

If you build it, they will came...

Basic experiment:

- large, rotating, $\sim$ QG (tilted caps) liquid metal or equivalent with $R_m \sim 100$

- extend domain of PPPL experiment of H. Ji, et.al.

- aims:
  - MHD dynamics of zonal flows, jets
  - zonal fields, QG dynamo (L-Smith, Tobias)
  - aspects of MHD momentum transport (i.e. solar tachocline physics)

‘Confinement’ experiments: (EAST?)

- high power, low τ_{ext} ITB

- multi-channel DBS imaging study of q_{min} region, on meso-scale

-multi-channel CHS to eliminate mean flows (including poloidal)

- high space-time resolution profile evolution, i.e. corrugations?

Conclusion

- There are no conclusions. This topic is alive and well, and will evolve dynamically.

- Cross-disciplinary dialogue with GFD/AFD communities has been very beneficial and should continue!